Public Good Provision and Optimal Taxation in a Hidden Income World

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Motivation

- According to World Bank Data, in 2018 the average of tax collection as a percentage of GDP in developing economies was 18% while in developed economies was 30%.
- One explanation: developing economies have large informal sectors, in which enforcing taxes is more costly.
  - According to the International Labour Organization, in 2018 the average rate of informal employment in developing economies was 48%.
- Another issue that comes with informality is the free-rider problem: tax recollection is used in government spending which has to consider the entire population of the economy, not just the ones who pay taxes.

Motivation

- Why does informality arise?
  - Informality is usually defined as a situation in which workers perform economic activities that are not covered by formal arrangements.[1]
  - Informal workers lack access to social security systems, as well as financial institutions.
  - In this case, the government has no way of directly observing the income of informal workers, which makes taxation difficult.
  - One explanation of why informality exists is that joining the financial system is not that appealing in developing economies because it does not have good returns and involves paying taxes.
- What is the tax system that maximizes welfare in the presence of hidden income/informality? Is zero informality optimal?

[1] This is the definition followed by the ILO, IMF, among others.

My Proposal

- In order to answer these questions I propose a hidden income model with the following ingredients:
  - The government can only observe the amount saved by agents in the financial system of the economy (namely, banks).
  - Agents live for two periods (young/old) and when young must decide how much to save for retirement, either in the bank or in the couch.
  - Taxes are collected from agents when young and they are used to construct a public good that will benefit consumers when old.
- The main trade-off the government faces when designing the tax system is that higher taxes push people out of the financial system, which in turn generates them a lower income when old, due to an underprovision of the public good.

Main Insights

- In the absence of income frictions, the welfare-maximizing tax system is progressive, and every individual participates in the financial system.

- In the presence of hidden income, the optimal tax schedule is concave, with zero marginal tax-rates at the top, and not necesarilly every individual in the economy participates in the financial system.
    - However, it is optimal (from a welfare perspective) to have every individual in the economy that participates in the financial system to not use other saving methods, i.e., zero evasion.

Related Literature

- Mirrlees Taxation: Kocherlakota (2005, 2010), Farhi and Gabaix (2020), Heathcote and Tsujiyama (2021).
- Hidden Income: Cole and Kocherlakota (2001), Doepke and Townsend (2006), Basetto and Phelan (2008), Vairo (2023).
- Public Goods Provision with Frictions: Mailath and Postlewaite (1990), Posner and Weinstein (2007), Bierbrauer (2009).

Basic Ingredients

- I consider an economy with a continuum of agents, represented by the $[0, 1]$ interval.
- Agents live for two periods:
  - Young: period in which they receive an income, must pay taxes, and decide how much to save for the future.
  - Old: period in which agents consume using their savings as well as the provision of a public good.
- Each agent will be characterized by an exogenously given type e, its income.
  - Agents receive their income when young i.i.d. from other agents.
  - Income has a finite support $E = \{e_1, ..., e_n\}$ with $e_1 < ... < e_n$.
  - The probability that agent $i \in [0, 1]$ has an income e_j is $\pi_j > 0$.

Basic Ingredients: Savings

- When young, agents may decide to save in two ways:
  - They can save in the bank, which pays a gross interest rate $R > 1$.
  - They can save in the couch, which pays an interest rate of 1.
- Savings (plus interest) are paid when old, which the agent uses to consume.

Basic Ingredients: Taxes and Public Good

- An additional consumption source for old agents is the provision of a public good.
  - This can be thought as a health system whose benefits are enjoyed by all old agents in the economy.
- To finanace this public good, taxes are recollected from each agent in the economy.
- The total amount of public good provided by the government, denoted g, will be the (weighted) average of agent's contributions.

Basic Ingredients: Observed/Hidden Productivity

- I am interested in studying the optimal tax sequence in this environment considering two scenarios:
  1. **Benchmark**: The government can perfectly observe e for every agent.
  2. The government does not know the income of any agent, but can observe how much the agent is saving.
- In both cases, the government's objective will be to maximize welfare in the economy.

Benchmark Model

- The government can perfectly observe the income of each agent.
- The government chooses a tax schedule $\tau : E \to \mathbb{R}$.
- Therefore, the utility of each agent that has income e_j is:

$$u\left(e_j - \tau(e_j) - b - b^c\right) + \beta u\left(Rb + b^c + \psi g\right),$$

  where $b, b^c \geq 0$ represent the amount of bank/couch savings; and $\psi > 1$ represents the positive externality that the public good has on the economy.
- I focus on the case $R \leq \psi$.
- The amount of public good provided is:

$$g = R \sum_{j=1}^{n} \tau(e_j)\pi_j.$$

Benchmark Model: Timing

- Timing in the model works as follows:
    1. The government **commits** to a tax schedule $\tau : E \rightarrow \mathbb{R}$ seeking to maximize welfare in the economy.
    2. Nature assigns each agent an income type according to F.
    3. Upon observing τ, g each agent chooses how much to save, denoted $b(e_j), b^c(e_j)$, to maximize her utility.
- Importantly, I am doing a "partial equilibrium" analysis, since the interest rate R is fixed, and I am not imposing market clearing on savings.

Benchmark Model

Government's Problem

The government solves the following problem:

$$\max_{\tau:E\to\mathbb{R}} \sum_{k=1}^{n} \left[u\left(e_k-\tau(e_k)-b(e_k)-b^c(e_k)\right)+\beta u\left(Rb(e_k)+b^c(e_k)+\psi g\right)\right]\pi_k,$$

$$(b(e_k),b^c(e_k)) \in \operatorname*{argmax}_{b,b^c\geq 0} \; u\left(e_k-\tau(e_k)-b-b^c\right)+\beta u\left(Rb+b^c+\psi g\right) \quad \forall \;\; e_k\in E,$$

$$g=R\sum_{j=1}^{n}\tau(e_j)\pi_j.$$

Benchmark Model: Savings and Consumption Decisions

Lemma

Let τ be fixed (and hence g). If $\beta R = 1$ and u is strictly concave, then:

1. There is a unique $(b(e_i), 0)$ that solves agent e_i's problem.
2. The optimal amount of bank savings are:

$$b(e_i) = \max\left\{\frac{e_i - \tau(e_i) - \psi g}{1+R}, 0\right\}.$$

3. Each agent with positive savings consumes the same amount when young and old (although this amount may vary across agents with different incomes).

Benchmark Model: Savings and Consumption Decisions

Definition

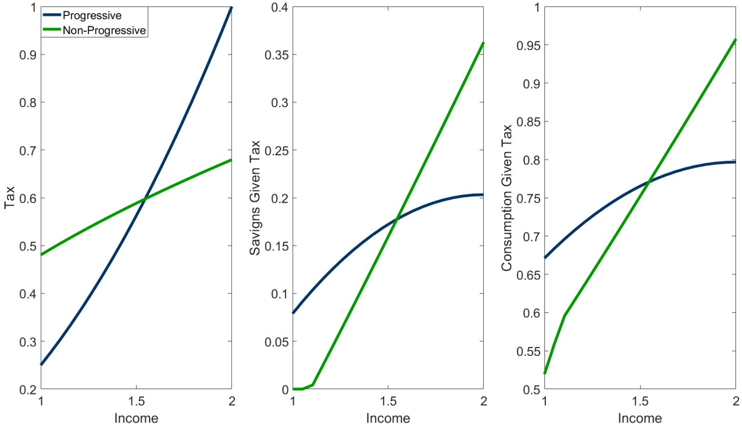
Given a tax schedule τ, the marginal tax rate for $e \in \{e_2, ...e_n\}$ is:

$$MTR(e_i) = \frac{\tau(e_i) - \tau(e_{i-1})}{e_i - e_{i-1}}.$$

A tax schedule is **progressive** if the marginal tax rate is non-decreasing in e_i.

- The shape of the tax schedule determines the savings of each agent, and hence its welfare.

Benchmark Model: Savings and Consumption Decisions



Note: This graph considers $R = 1.05$, $E \sim U([0, 1])$, and $\psi = 1.2$.

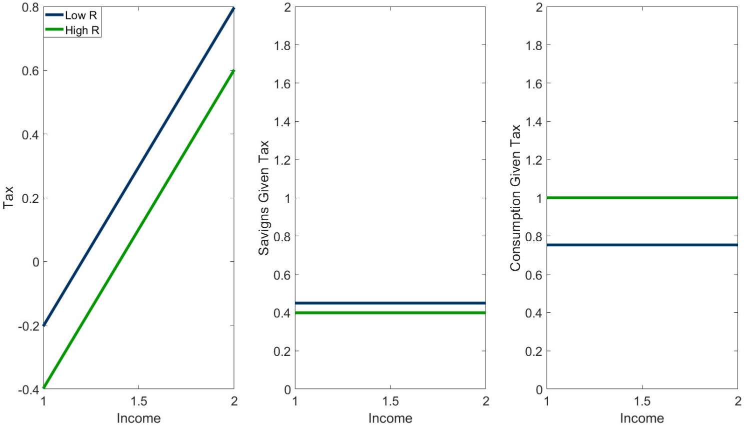
Benchmark Model: Optimal Tax Schedule

Proposition

If $\beta R = 1$ and u is strictly concave, then:

1. The optimal tax schedule $\tau^{\star}$ equates consumption across all agents with positive savings.
2. The optimal tax schedule $\tau^{\star}$ equates consumption when young across all agents.
3. If all agents have positive savings given $\tau^{\star}$, then the optimal tax schedule $\tau^{\star}$ is progressive.

Benchmark Model: Comparative Statics in R



Note: This graph considers $R_L = 1.01$, $R_H = 1.1$, $E \sim U([0,1])$, and $\psi = 1.2$.

Hidden Income Model

- Now, I study a model in which the government can no longer directly observe agent's income.
- The government can only observe the amount of each agent's savings in the financial system.
- Therefore, the tax schedule is a function $\tau : \mathbb{R} \to \mathbb{R}$ where the domain represents the savings that each agent makes in the bank.
- The utility of a type e agent is given by:

$$u\left(e - b - b^c - \tau(b)\right) + \beta u\left(Rb + b^c + \psi g\right),$$

where b is the amount of savings the agent has deposited in the bank, and b^c is the amount of couch savings.

Hidden Income Model: Timing

- Timing in the model works as follows:
  1. The government **commits** to a tax schedule $\tau : \mathbb{R} \to \mathbb{R}$ seeking to maximize welfare in the economy.
  2. Nature assigns each agent an income type according to F.
  3. Upon observing τ, g each agent chooses how much to save, denoted $b(e_j), b^c(e_j)$, to maximize her utility.

Hidden Income Model

- What changes with respect to the full information framework?

1. The shape of the tax schedule becomes relevant for every agent, since now tax payments depend on the amount of indivudual savings.
2. Essentially, this becomes a public good contribution game (with τ fixed).
3. Couch savings represent an outside option for agents, so the government needs to consider that higher taxes may push people towards the couch (which I interpret as evasion).

Definition

An agent is **informal** if $b = 0$ and $b^c \geq 0$, while an agent is an **evader** if $b > 0$ and $b^c > 0$.

Hidden Income Model: Equilibrium Given τ

Definition

Let τ be fixed. A (Nash) equilibrium outcome g_τ is a value such that there are $\{b(e), b^c(e)\}$ with the property that:

1. Given τ, g_τ; $(b(e), b^c(e))$ solve:

$$\max_{b^c \geq 0, b} \; u\left(e - b - b^c - \tau(b)\right) + \beta u\left(Rb + b^c + \psi g_\tau\right),$$

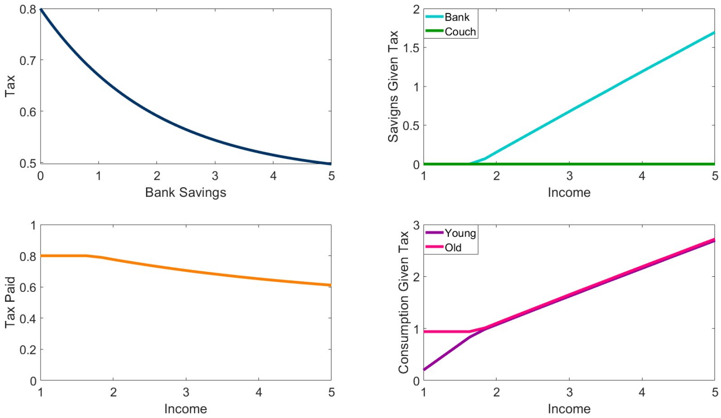
   for every $e \in E$.

2. $$g_\tau = R \sum_{j=1}^{n} \tau(b(e_j))\pi_j.$$

Let $\mathcal{G}(\tau)$ be the set of all possible equilibrium outcomes given τ.

Hidden Income Model: Equilibrium Given τ

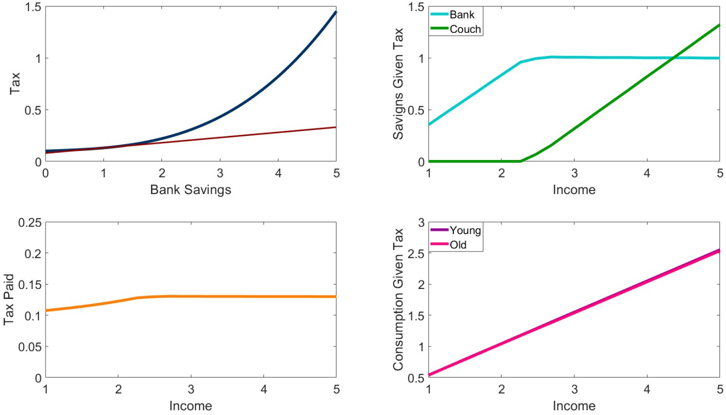
$$g_\tau = 0.71.$$



Note: This graph considers $R = 1.05$, $E \sim U([0, 5])$, and $\psi = 1.25$.

Hidden Income Model: Equilibrium Given τ

$$g_\tau = 0.12.$$



Note: This graph considers $R = 1.05$, $E \sim U([0, 5])$, and $\psi = 1.25$.

Hidden Income Model: Equilibrium Given τ

Observation

Let τ be fixed.

1. If $\mathcal{G}(\tau) \neq \emptyset$ and $\beta R = 1$:
   - The optimal savings imply that each agent consumes the same amount when young and old (although this amount may vary across agents with different incomes).
2. Unless some structure is imposed on τ, there is no guarantee for $\mathcal{G}(\tau) \neq \emptyset$, even under $\beta R = 1$ and u being strictly concave.
3. I restict attention to $\tau \in \mathcal{T}$ which is the set of continuous, bounded functions that also have a continuous and uniformly bounded first derivative.

Hidden Income Model: Government's Problem

Challenges

Government's Problem

The government solves the following problem:

$$\max_{\tau \in \mathcal{T}} \sum_{k=1}^{n} \left[u\left(e_k - b(e_k) - b^c(e_k) - \tau\left(b(e_k)\right)\right) + \beta u\left(Rb(e_k) + b^c(e_k) + \psi g\right)\right] \pi_k,$$

$$g \in \mathcal{G}(\tau).$$

Hidden Income Model: Optimal Tax Schedule

Optimal Tax Conjeture

If $\beta R = 1$ and u is strictly concave:

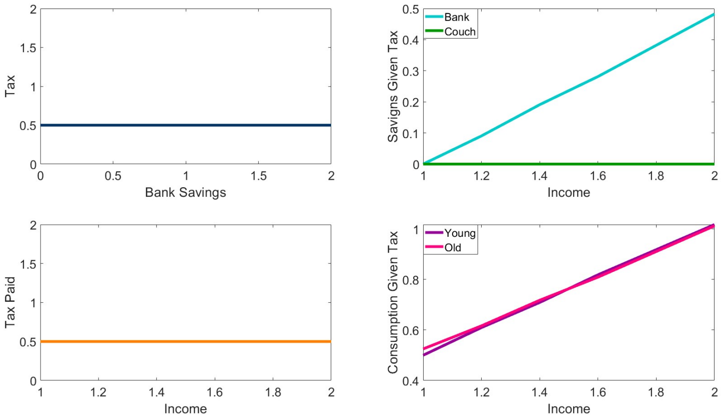
1. Any strictly increasing and convex tax schedule will not be optimal.
2. There exists a region $\mathcal{B}$ such that $\tau^\star(b) = \bar{\tau}$ for all $b \in \mathcal{B}$.
3. The optimal tax $\tau^\star$ implies zero evasion for every agent such that $b(e_j) \neq 0$.
4. The optimal tax $\tau^\star$ does not necesarilly imply zero informality.
5. The optimal tax $\tau^\star$ does not guarantee equal consumption **across** agents.

Hidden Income Model

- The government wants to erradicate evasion, since this decreases both the amount of public good provided as well as consumption when old.
- To do this, $\tau(b) \leq R$ for all b.
- When interest rates are low, evading becomes "more atractive", and hence the government cannot charge high taxes.
- **Economies with low returns on the financial system will then have both lower tax recollection and lower public good provision.**

Hidden Income Model: Optimal Taxes with Low Interest Rate

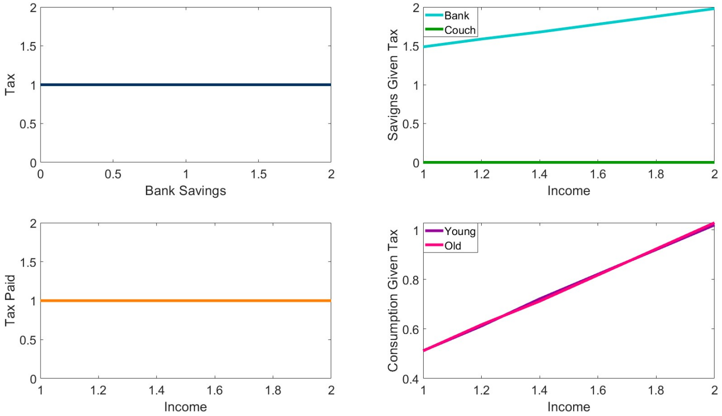
$g_{\tau^\star} = 0.5$, Informality Rate > 0, Evasion Rate $= 0$.



Note: This graph considers $R = 1.01$, $E \sim U([0,1])$, and $\psi = 1.2$.

Hidden Income Model: Optimal Taxes with High Interest Rate

$g_{\tau^\star} = 1.01$, Informality Rate $= 0$, Evasion Rate $= 0$.



Note: This graph considers $R = 1.1$, $E \sim U([0,1])$, and $\psi = 1.2$.

Hidden Income Model

- Currently "lump-sum" taxes seem to be optimal, since I am restricting $\mathcal{T}$ to be the set of continuous, differentiable functions.
- Using first-order conditions, one can show that a flat region is required in the tax schedule to avoid evasion.
- **Current Work:** non-decreasing step functions lead to higher welfare and public good provision.
  - So, taxing more to those agents who save more may be optimal, but it has to be done in a way that does not push agents to evade.
  - Preliminary results that more bins with higher taxes are optimal as R increases.

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Hidden Income Model: Challenges Back

- Assuming that now income is hidden and taxation can be done via savings poses some challenges.
- **Theoretical:**
  1. Government's problem is now a **variational problem** in which the tax schedule decision affects not only individual decisions, but also the set of equilibria that can be implemented.
  2. Existance requires to show: (i) that the correspondance $\mathcal{G}$ is continuous in τ; (ii) Euler-Lagrange conditions are satisfied.
- **Numerical:**
  1. Need to approximate the problem, using for example Ritz's Method.
  2. Show that this approximation converges to the actual solution of the original problem.