Debt, Inflation, And Government Reputation

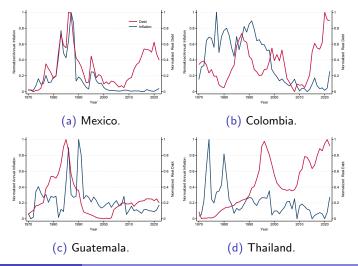
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Motivation

• During the 20th century, many emerging economies experienced high debt levels that, in some episodes, were followed by high inflation; while in others, inflation was less responsive.



Motivation

- Of course, many of these countries experienced a structural change, Central Bank autonomy, which contributed to control inflation and disconect it from debt.
- Nevertheless, the literature (Kocherlakota, 2012; Bassetto and Miller 2023) suggests that this disconnection between inflation and debt is not automatic.
- Agents' perception on how committed is the government to low inflation is crucial to determine this relationship.
- In this paper, I propose an incomplete information game between private agents and the government, in which government reputation is key to determine inflation and debt dynamics.
- Government Reputation: probability agents assign to be facing a governmnet that is committed to low inflation.

My Proposal

- I consider a game between private agents and a **consolidated government**, which makes decisions on both inflation and debt.
- Building on Kydland and Prescott (1977) and Barro and Gordon (1983), in my model:
 - There are wage setters that choose wages each period, aiming to have a constant real wage over time.
 - On the government chooses money growth and debt, aiming to peg output, inflation and debt to a target.
- Main tension: a time inconsistency problem between the decisions of wage setters and the government, which may be worsen by the debt state the economy is in.

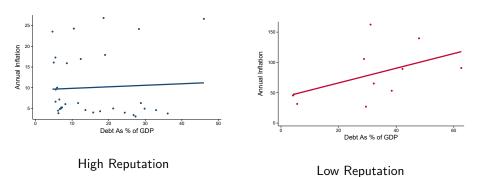
Time Inconsistency Problem

- Main idea: the government is incentivized to generate a higher inflation in order to stimulate output.
- How to mitigate this time inconsistency problem?
- The literature has provided several answers to this question:
 - 1 Limit the discretion of the Central Bank.
 - ★ Rules: Taylor (1983), Halac and Yared (2014), Dovis and Kirpalani (2021).
 - * Inflation Caps: Athey, Atkenson, and Kehoe (2005).
 - 2 Repeated Games: Barro and Gordon (1983), Backus and Driffill (1985).
 - Reputation approach : Phelan (2006), Lu et al. (2016), Dovis and Kirpalani (2021).
- My research question in this context: how does the interaction of debt and government reputation mitigate/worsen this time inconsistency problem?

Main Findings

- Both debt and reputation are crucial to determine government behavior.
- As debt increases, incentives to generate higher inflation increase.
- However, a government with high reputation reacts less severely.
- "Low Reputation Effect": even a government committed to low inflation may choose elevated inflation and deficits when it has low reputation, since agents expect to be facing a government with less committment.
- An increasing debt sequence can lead to a deterioration in government reputation, if it is followed by high inflation; or a gain in government reputation, if it is followed by low and controlled inflation.
- All this behavior is captured within a single equilibrium with no changing types.

Main Findings Model and Data



• In my model, inflation will be highly correlated with public debt when government reputation is low, and less correlated when reputation is high.

Model: Stage Game

- Two players: wage setters, and a government.
- Time is discrete *t* = 1, 2, 3, ...
- In each period, both players choose their actions simultaneously:
 - **1** Wage setters choose w_t .
 - **2** The government chooses inflation (π_t) and deficit (d_t) .
- The choice of these variables determines the price level p_t , output y_t , and the evolution of debt in the economy b_t .

Model: Stage Game

• In each period:

• Wage setters choose w_t which pins down π_t^e : WS Problem

$$UW_t = -\left(\frac{w_t - p_t}{p_{t-1}}\right)^2 = -(\pi_t - \pi_t^e)^2.$$

• Government chooses π_t, d_t : G Problem

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2,$$

where:

$$y_t = ar{y} + heta(\pi_t - \pi_t^e) + d_t,$$

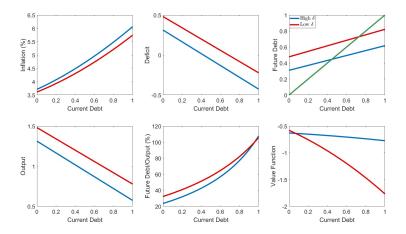
 $b_t = d_t + rac{(1+r)(1+\pi_t^e)b_{t-1}}{1+\pi_t} - ar{m}\pi_t.$

Model: Dynamic Game

- I first analyze the behavior of players if they were to interact repeatedly over time.
- Complete information framework: no reputational concerns.
- This is a dynamic game, since debt is a state variable.
- Since there are no intertemporal considerations in the wage setters' problem, they choose their wage myopically.
- The government chooses inflation and debt taking into account future implications of its decisions.
 - The government discounts the future with factor $\delta \in [0, 1)$.
- Perfect monitoring structure: at each *t*, players observe the history of the game up to that point.
- I focus on Markov perfect equilibria. Markov Equilibrium Definition

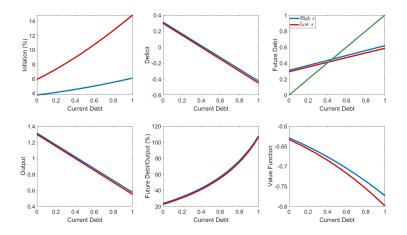
Equilibrium Example: The Role of δ

$$V(b) = \max_{\pi,d,b'} (1-\delta) \left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2 \right] + \delta V(b').$$



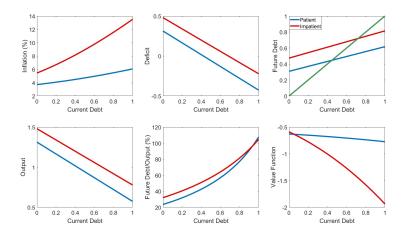
Equilibrium Example: The Role of s

$$V(b) = \max_{\pi,d,b'} (1-\delta) \left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2 \right] + \delta V(b').$$



Equilibrium Example: Patient vs Impatient Governments

$$V(b) = \max_{\pi, d, b'} (1 - \delta) \left[-(y - k\bar{y})^2 - s(\pi - \bar{\pi})^2 - \gamma(b')^2 \right] + \delta V(b').$$



Takeayaws of Dynamic Game

- A government with higher discount factor, δ , generates a lower debt policy function and a lower long-run debt level.
- A government with higher disutility for inflation, *s*, generates a lower inflation policy function and a lower long-run inflation level.
- This model can explain, by varying δ and s, how come a government could generate high inflation with high debt levels, or low inflation with low debt levels.
- However, the model has problems in explaining why a government would generate low inflation with high debt levels.
- Solution: introduce reputational concerns.

Reputations Framework Details

- In this setting, wage setters now interact with a government that may be of two types:
 - Type *P* (Patient): a government that has a discount factor $\delta_P \in (0, 1)$ and a disutility for inflation parameter s_P .
 - Orrection 2 Type I (Impatient): a government that has a discount factor 0 ≤ δ_I < δ_P and s_I < s_P.
- Let $\rho_0 \in [0,1]$ be the prior probability that the government is of type *P*.
- Players will observe a noisy signal of the government's actions:

$$ilde{\pi}=\pi^{\xi}+\epsilon_{\pi}, \ \ ilde{d}=d^{\xi}+\epsilon_{d}, \ \ ilde{b}'= ilde{d}+rac{(1+r)(1+\pi^{e})b}{1+ ilde{\pi}}-ar{m} ilde{\pi},$$

where ϵ_x are i.i.d. random variables with mean zero and variance σ_x^2 .

Reputations Framework: Equilibrium Graph

Theorem

A perfect Markov separating equilibrium of this repeated game exists. Furthermore, in any equilibrium it must be the case that:

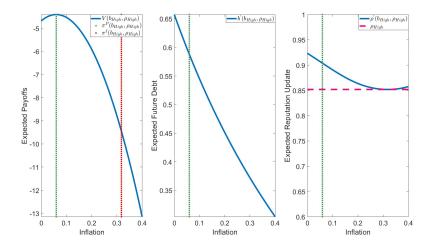
$$\pi^{\mathsf{e}}(b,\rho) = \rho \pi^{\mathsf{P}}(b,\rho) + (1-\rho)\pi^{\mathsf{I}}(b,\rho) \quad \text{for all } b \in [0,\bar{b}], \ \ \rho \in [0,1].$$

In this equilibrium, it is the case that for all (b, ρ) :

$$\frac{\partial \pi^{e}}{\partial b}(b,\rho) > 0, \quad \frac{\partial \pi^{e}}{\partial \rho}(b,\rho) < 0, \quad \frac{\partial^{2} \pi^{e}}{\partial b \partial \rho} < 0.$$

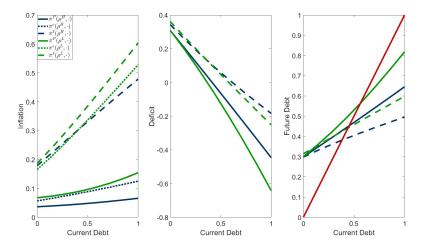
Reputations Framework: Equilibrium

• In this game, the patient government faces a trade-off between either increasing reputation or decreasing future debt.



Equilibrium: High vs Low Reputation

• As government reputation decreases, the patient government has incentive to generate higher inflation and debt.



Equilibrium: High vs Low Reputation

- Why does a patient government choose high inflation and deficit levels when it has low reputation?
- In equilibrium, whenever ρ is low, inflation expectations are close to $\pi^{I}(b, \rho)$.
- Hence, if the patient government were to choose an inflation level that is considerably lower than $\pi^{l}(b, \rho)$, this would generate both a lower output and a higher debt level than if it were to choose something close to $\pi^{l}(b, \rho)$:

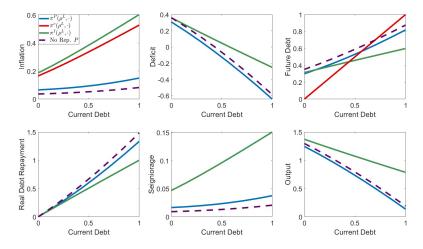
$$y = \bar{y} + \theta \left(\pi^{P}(b,\rho) - \pi^{e}(b,\rho) \right) + d^{P},$$

$$b' = d^{P} + \frac{(1+r)(1+\pi^{e}(b,\rho))b_{t}}{1+\pi^{P}(b,\rho)} - \bar{m}\pi^{P}(b,\rho).$$

• In order for the patient government to have reputation gains, it would need to choose an inflation level that is considerably lower than $\pi'(b, \rho)$, which is not optimal.

Equilibrium: Low Reputation

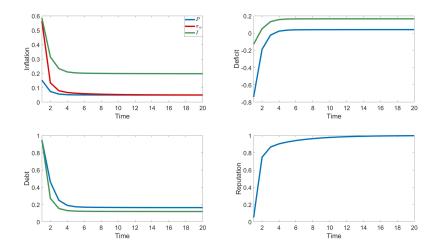
 If the patient government had no reputation concerns, it would choose lower inflation, which would lead to higher debt, higher real interest rates, and lower seigniorage.



Long-Run Learning

- Upon observing noisy realization of the government's actions, wage setters update their beliefs on the government's type.
- Since equilibrium generates stationary time series for inflation and debt, wage setters will learn the government's type in the long-run.
- This results goes in line with known results in reputations literature (e.g., Cripps, Mailath, and Samuelson, 2004).

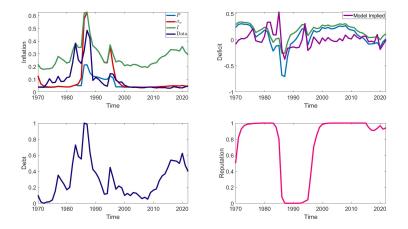
Equilibrium: Long-Run Learning (Learning with Noise)



Model Predictions and Mexican Data Details

- The previous results presented are from a model calibrated to match some features of the Mexican data between 1970-2022. Parameter Values
- Now, I present the predicted time series for government reputation, inflation expectations, and fiscal deficit according to the model, using as input the data time series of inflation and debt.
- I considered an initial value of debt of $b_0 = 0.1$ and a prior probability of the government being patient of $\rho_0 = 0.5$.

Model Predictions and Mexican Data Details Scatter Plot



Key Takeaways of Reputations Model

- The value of reputation is crucial to determine government behavior.
 - Even a patient government finds it optimal to choose high inflation and deficits when it has low reputation.
 - As its reputation increases, we should expect a disconnection between inflation and debt.
- The model predicts that inflation expectations capture the government's reputation.
 - If we observe an increase in debt and in expectations, this could be a signal of a deterioration in the government's reputation.

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Model: Wage Setters

- I model the labor market as a monopolistic competition market, in which there is a continuum of wage setters, indexed by *i* ∈ [0, 1].
- Wage setter *i* chooses w_t^i having as an objective a constant real wage over time:

$$UW_t^i = -\left(rac{w_t^i - p_t}{p_{t-1}}
ight)^2.$$

- Each *i* knows their wage decision does not affect p_t , and hence their expected utility maximizing wage choice is *i*'s expected price level, $w_t^i = p_t^{e,i}$.
- Since, by assumption, every wage setter has the same information when deciding w_t , then $w_t = p_t^{e,i} = p_t^e$ for all *i*.

Model: Wage Setters Back

• If we define inflation as $\pi_t = (p_t - p_{t-1})/p_{t-1}$, and $\pi_t^e = (p_t^e - p_{t-1})/p_{t-1}$ then we can re-write the wage setters payoffs as:

$$UW_t^i = -\left(\frac{w_t^i - p_t}{p_{t-1}}\right)^2 = -\left(\frac{p_t^e - p_{t-1} + p_{t-1} - p_t}{p_{t-1}}\right)^2 = -(\pi_t^e - \pi_t)^2.$$

 From now on, I consider π^e_t to be the relevant variable for wage setters, which is pin downed by their wage decision.

Model: Government Back

- The government makes decisions on both the fiscal and monetary aspects of the economy.
- Each period, the government inherits a debt level b_{t-1} , and must decide on the deficit level d_t (government expenditures minus income) and growth rate of money g_t .
- The government is interested in pegging output, inflation, and debt to a target level:

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2,$$

where $k > 1, s > 0, \gamma > 0$, \bar{y} is the natural output level, and $\bar{\pi}$ is the inflation target of the government.

Model: Output Back

- Following the Neo-Keynesian tradition, output varies around a natural level \bar{y} .
- These fluctuations are driven by the labor market.
 - In an economy with sticky wages, higher prices attract more firms and workers to the market, increasing output.
- Hence, output is given by:

$$y_t = \bar{y} + \theta(p_t - w_t) + d_t = \bar{y} + \theta(\pi_t - \pi_t^e) + d_t,$$

where $\theta > 0$ measures the effect of the labor market on output.

Model: Evolution of Debt Back

- The evolution of government debt will be determined by:
 - Real (primary) deficit d_t: the difference between government expenditures and income.
 - Real service of debt: the amount of previous debt that the government must pay.
 - Seigniorage: the revenue that the government gets from printing money.
- Importantly, according to the Fisher equation, the interest rate that the government faces is:

$$1 + i_t = (1 + r)(1 + \pi_t^e),$$

where r > -1 is the natural interest rate.

Model: Evolution of Debt Back

• The evolution of debt in real terms is given by:

$$b_t = d_t + rac{(1+i_t)b_{t-1}}{1+\pi_t} - S_t = d_t + rac{(1+r)(1+\pi_t^e)b_{t-1}}{1+\pi_t} - S_t.$$

• Then, higher inflation (as a result of higher money growth) generates a lower real interest rate and hence a higher seigniorage, which in turn will reduces the real value of debt.

Model: Restrictions on Debt Back

- I restrict debt to be non-negative, i.e., $b_t \ge 0$.
- Also, I assume that the government has a bound on the amount of debt it can issue $\bar{b}.$ Parameter Restrictions
- I focus on the case \bar{b} is large and not-binding, although future work where \bar{b} is binding could be interesting.
- Let $\mathcal{D} = [0, \overline{b}]$ be the set of feasible debt levels.

Model: Real Balances Demand and Money Growth Back

- To close up the model, I need to specify how prices are determined.
- Again, following the Neo-Keynesian tradition, I assume that prices and money are related through a money demand function.
- To keep the model simple, I assume that the government faces the following money demand function:

$$\frac{m_t}{p_t} = \bar{m}_t$$

which has two implications:

- **(** $g_t \pi_t = 0$, i.e., by choosing g_t the government is pinning down inflation.
- 2 Seigniorage is then given by $S_t = \bar{m}\pi_t$.

Model: Markov Strategies Back

- I make a restriction to stationary Markov pure strategies.
 - Wage setters choose a strategy π^e : ℝ → ℝ, where π^e(b) represents inflation expectations upon observing a previous debt level b.
 - The government chooses $\sigma_g: \mathbb{R} \to \mathbb{R}^3$, where

 $(\pi(b),d(b),b'(b))$

represent the inflation, deficit, and debt decisions upon observing b and given the strategy σ_w of wage setters.

• I restrict the strategy of wage setters to be a concave and twice differentiable function with uniformly bounded first derivatives.

Dynamic Game: Government's Problem Back

• Given the current debt b and a conjecture on government inflation behavior $\hat{\pi}$, the wage setters best reply is such that:

$$\pi^e(b) = \operatorname*{argmax}_{\pi^e} - (\pi^e - \hat{\pi}(b))^2.$$

Dynamic Game: Government's Problem (Back)

• Given the current debt b and a conjecture on wage setters' behavior $\hat{\sigma}_w$, the government's best reply is the solution of the following problem:

$$V(b) = \max_{\pi,d,b'} \quad (1-\delta) \left[-(y-k\bar{y})^2 - s(\pi-\pi^*)^2 - \gamma(b')^2 \right] + \delta V(b')$$
$$y = \bar{y} + \theta \left(\pi - \hat{\pi}^e(b)\right) + d,$$
$$b' = d + \frac{(1+r)(1+\hat{\pi}^e(b))b}{1+\pi} - \bar{m}\pi,$$
$$0 \le b' \le \bar{b}.$$



Equilibrium

A Markov perfect equilibrium of this dynamic game is a strategy profile (π^e, σ_g) such that:

- **1** (π^e, σ_g) are stationary Markov strategies.
- 2 Taking π^e as given, σ_g solves the problem described in the previous slide.
- **3** Taking σ_g as given, wage setters find π^e to maximize their payoffs.

Theorem*

A Markov Perfect Equilibrium of this dynamic game exists. In this equilibrium:

 $\pi^{e\star}(b) = \pi^{\star}(b).$

Furthermore, the following properties hold:

- **0** V^* is a concave function of current debt $b \in [0, \overline{b}]$.
- **2** $\pi^{e\star}$ is an increasing function of $b \in [0, \overline{b}]$.
- **3** $\pi^{\star}(b)$ is an increasing and differentiable function of $b \in [0, \overline{b}]$.
- **(**) $d^{\star}(b)$ is a decreasing and differentiable function of $b \in [0, \overline{b}]$.
- **3** Let s > s'. Then, $\pi^*(b|s) \le \pi^*(b|s')$ for all $b \in [0, \overline{b}]$.
- Let k > k'. Then, $\pi^*(b|k) \ge \pi^*(b|k')$ for all $b \in [0, \overline{b}]$.

Parameter Restrictions

• For the restriction $b' \leq \bar{b}$ to be not binding, I need to impose some parameter restrictions.

Assumption

Let $\bar{y} = 1$. Then, for $b' \leq \bar{b}$ in the stage game for all possible values of $b \in D$, the parameters must satisfy:

$$rar{b} \leq rac{ar{b}\gamma(r(s-1)-s)+\gamma k}{\gamma(1+s+ heta)+s}.$$

All the examples presented in this presentation satisfy this restriction. Back

Dynamic Game: Equilibrium

- Given σ_w , the government's (inflation) best reply is π .
- Under concavity and differentiability assumptions on σ_w , the government's best reply is unique.
- Therefore, the government's problem creates a mapping $\sigma_w \to \pi$.
- Given the utility of wage wetters $-(\pi \pi^e)^2$, an equilibrium of this game is a fixed point of such mapping.
- Existence of such fixed point is guaranteed by the Schauder Fixed-Point Theorem*.
- Equilibrium characterization can be done using the Implicit Function Theorem, the Envelope Theorem, and the Benveniste-Scheinkman Theorem.

The Role of γ

• Recall that the flow payoffs for the government are:

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2.$$

- The parameter γ captures the government's aversion to debt.
- Some have suggested to consider $\gamma = 0$ in order to simplify the analysis.
- However, this leads to an uninteresting equilibrium where the government chooses a constant inflation rate.

The Role of γ

Proposition

Suppose $\gamma = 0$ and $\bar{y}(1 + \bar{\pi} - k) \leq r\bar{b}$. Then, in the unique Markov equilibrium of the dynamic game $\pi(b) = \bar{\pi}$, $d(b) = (k - 1)\bar{y}$, and V(b) = 0 for all $b \in \mathcal{D}$.

The Role of γ

Proof

- Since $\pi^e(b) = \overline{\pi}$, then $y = \overline{y} + \theta(\pi \overline{\pi}) + d$.
- Then, since the government's flow payoffs are $-(y k\bar{y})^2 s(\pi \bar{\pi})^2$, the government will choose $\pi = \bar{\pi}$ and $d = (k 1)\bar{y}$.
- This gives the government a flow payoff of zero, which is the highest achievable flow payoff.
- In order for the Bellman equation to hold, the value function must be zero.
- Notice that in this case $b' = (k-1)\bar{y} + (1+r)b \bar{y}\bar{\pi}$, which converges to $b = \max\{0, \frac{\bar{y}(1+\pi-k)}{r}\} \le \bar{b}$ as long as $\bar{y}(1+\bar{\pi}-k) \le r\bar{b}$.
- Hence, the term $-\gamma b_t^2$ creates a trade-off between inflation and deficit, which is also impacted by the current debt level. Back

Reputations Framework: Timing Back

- At each period *t*, the game proceeds as follows:
 - Upon the previous history of play, wage setters form a belief on the type of government they are facing.
 - Both wage setters and the government choose their actions simultaneously.
 - **③** The shocks $(\epsilon_{\pi}, \epsilon_d)$ are realized, and wage setters observe $(\tilde{\pi}_t, \tilde{d}_t, \tilde{b}'_t)$.
 - The history of play for the next period will be given by $h^{t+1} = (h^t, \tilde{\pi}_t, \tilde{d}_t, \tilde{b}_t')$.

Reputations Framework: Markov Strategies (Back)

- Once again, I restrict attention to analyze pure Markov strategies.
- In this dynamic game, there are now two state variables:
 - The previous debt level b.
 - 2 Government reputation ρ , i.e., the belief of wage setters that the government is of type P given the observed history of play up until that point.

Reputations Framework: Updating Rule Back

• In equilibrium, upon observing $(\tilde{\pi}, \tilde{d})$, wage setters will update their beliefs according to Bayes' Rule:

$$\begin{array}{l} \rho\left(b,\rho\right) = \\ \\ \frac{\rho g_{\pi}\left(\tilde{\pi}-\pi^{P}(b,\rho)\right)g_{d}\left(\tilde{d}-d^{P}(b,\rho)\right)}{\rho g_{\pi}\left(\tilde{\pi}-\pi^{P}(b,\rho)\right)g_{d}\left(\tilde{d}-d^{P}(b,\rho)\right)+(1-\rho)g_{\pi}\left(\tilde{\pi}-\pi^{I}(b)\right)g_{d}\left(\tilde{d}-d^{I}(b,\rho)\right)}, \end{array}$$

Reputations Framework: Wage Setters Problem

Taking as given (b, ρ) and a conjecture on government behavior π^P, π^I, wage setters' best reply is characterized by the following problem:

$$\pi^{\mathbf{e}}(b,\rho) = \operatorname*{argmax}_{\pi^{\mathbf{e}}} \quad \mathbb{E}\left[-\rho(\pi^{\mathbf{e}} - \tilde{\pi}^{P}(b,\rho))^{2} - (1-\rho)(\pi^{\mathbf{e}} - \tilde{\pi}^{I}(b,\rho))^{2}\right],$$

where
$$ilde{\pi}^P(b,
ho) = \hat{\pi}^P(b,
ho) + \epsilon^P_\pi$$
 and $ilde{\pi}^I(b,
ho) = \hat{\pi}^I(b) + \epsilon^I_\pi$.

• Then, the best reply of wage setters is given by:

$$\pi^{e}(b,\rho) = \rho \hat{\pi}^{P}(b,\rho) + (1-\rho) \hat{\pi}^{I}(b,\rho).$$

Reputations Framework: Type P's Problem Back

• Taking as given (b, ρ) and a conjecture on wage setters $\hat{\sigma}_w$ as well as the behavior of the government of type I, the government of type P's best reply is characterized by the following problem:

VP(I)

$$\begin{split} \nabla^{-}(b,\rho) &= \\ \max_{\pi,d,b'} \quad \mathbb{E}_{\epsilon_{\pi},\epsilon_{d}} \left[(1-\delta) \left[-(\tilde{y}-k\bar{y})^{2} - s_{P}(\tilde{\pi}-\bar{\pi})^{2} - \gamma(\tilde{b}')^{2} \right] + \delta V^{P}(b',\rho') \right], \\ \tilde{y} &= \bar{y} + \theta \left(\tilde{\pi} - \hat{\pi}^{e}(b,\rho) \right) + \tilde{d}, \\ \tilde{b}' &= d + \frac{(1+r)(1+\hat{\pi}^{e}(b,\rho))b}{1+\tilde{\pi}} - \bar{m}\tilde{\pi}, \\ \tilde{b}' &= d + \epsilon_{d}, \\ 0 &\leq \tilde{b}' \leq \bar{b}. \end{split}$$
$$\rho' &= \frac{\rho g_{\pi} \left(\tilde{\pi} - \pi \right) g_{d} \left(\tilde{d} - d \right)}{\rho g_{\pi} \left(\tilde{\pi} - \pi \right) g_{d} \left(\tilde{d} - d \right) + (1-\rho) g_{\pi} \left(\tilde{\pi} - \hat{\pi}' \right) g_{d} \left(\tilde{d} - d' \right)}. \end{split}$$

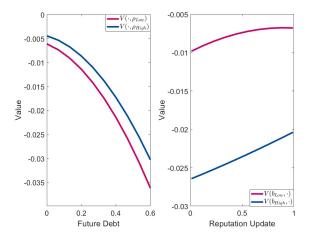
π

Reputations Framework: Type I's Problem Back

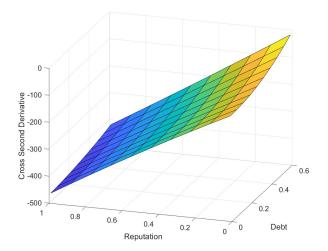
• Taking as given (b, ρ) and a conjecture on wage setters $\hat{\sigma}_w$, the government of type *I*'s best reply is characterized by the following problem:

$$egin{aligned} \mathcal{W}^{I}\left(b,
ho
ight)&=\max_{\pi,d,b'}\quad \mathbb{E}_{\epsilon_{\pi},\epsilon_{d}}\left[-(ilde{y}-kar{y})^{2}-s_{I}(ilde{\pi}-ar{\pi})^{2}-\gamma(ilde{b}')^{2}
ight],\ & ilde{y}&=ar{y}+ heta\left(ilde{\pi}-ar{\pi}^{e}(b,
ho)
ight)+ ilde{d},\ & ilde{b}'&=d+rac{(1+r)(1+ar{\pi}^{e}(b,
ho))b}{1+ar{\pi}}-ar{m}ar{\pi},\ & ilde{\pi}&=\pi+\epsilon_{\pi},\ & ilde{d}&=d+\epsilon_{d}.\ & ilde{0}&\leqar{b}. \end{aligned}$$

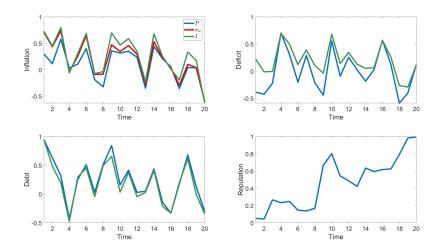
Value Function Properties



Value Function Properties (Back)



Equilibrium: Long-Run Learning Back



Model Predictions and Mexican Data Back

- I considered the time series data $(\pi_t^{data}, b_t^{data})$ for Mexico between 1970-2022.
- Then, using (b_0, ρ_1) as initial conditions, I computed the time series for $(\pi_t^P, \pi_t^I, \pi_t^e, \rho_t, d_t)$ according to the model.
- For the fiscal deficit, I computed the fiscal deficit that is consistent with the model:

$$d_t = b_t^{ extsf{data}} + ar{m} \pi_t^{ extsf{data}} - rac{(1+r)(1+\pi_t^e)b_{t-1}^{ extsf{data}}}{1+\pi_t^{ extsf{data}}},$$

• For the reputation update, I considered the likelikood of receiving shocks of size $\pi_t^{data} - \pi_t^P$ vs $\pi_t^{data} - \pi_t^I$.

Parameter Values for Mexico 1970-2022 (Back

Parameter	Interpretation	Value
Ţ	Natural Level of Output	1
θ	Sensitivity of Output to Inflation	0.5
k	Time Inconsistency Parameter	2
γ	Debt Weight	2
$\overline{\pi}$	Inflation Target	3%
r	Interest Rate	5%
SP	Inflation Target Weight Patient Government	80
SI	Inflation Target Weight Impatient Government	5
δ_P	Discount Factor Patient Government	0.45
δ_I	Discount Factor Impatient Government	0