A Stochastic Choice Model from Lists with Thresholds*

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Abstract

Many real-world choice environments (a shopper scrolling through Amazon, a voter reading down a ballot, a user swiping through a dating app) present a fixed set of alternatives in an order that varies, rather than varying which alternatives are on offer. I study a model of choice from such lists in which the decision maker searches in order and stops at the first alternative that is at least as good as a threshold she has in mind, provided this happens before she exhausts a limited search capacity. I first study a deterministic version of this procedure, in which the threshold and search capacity are fixed. I show that, although its primitives can only be partially identified from behavior, the choice functions it generates admit a complete axiomatic characterization. I then randomize both primitives and show that this Random Depth Random Threshold Model resolves the deterministic model's identification problem: the underlying preference is always identified, and, under an independence condition on the joint distribution of search depth and threshold, so is the full joint distribution itself. I propose six axioms on stochastic choice from lists and show that they are necessary and sufficient.

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1 Introduction

Many of the choice environments people face today do not vary in which alternatives are available, but in the order in which those alternatives are presented: a shopper scrolling through Amazon's search results, a voter reading down a ballot, and a user swiping through a dating app all choose from the same underlying set of alternatives, encountered in a sequence that depends on the platform, the search query, or simply the order of presentation. A ballot fits this description exactly, since the same candidates appear in every printing of it; Amazon's search results and a dating app's queue, by contrast, often vary the underlying set of alternatives as well, as new listings appear or old ones sell out. Still, conditional on a fixed result set or candidate pool, the ranking algorithm behind either platform is free to vary the order in which that same set is displayed from one user, session, or query to the next, which is the feature this paper isolates and the reason list, rather than menu, variation is the object of study below. A large empirical literature documents that this order matters for choice. As argued by Horan (2010), the prevailing view in psychology and marketing is that order effects bias decision-making. The *serial position effect*, for example, suggests that agents remember and choose alternatives placed at the end of a list, while *anchoring* suggests that the first alternative disproportionately shapes the choice that follows.

This paper develops a model of choice from lists that captures order effects through a specific, portable mechanism: the decision maker searches a list in the order it is presented and stops at the first alternative that clears a threshold she has in mind, provided she reaches it before exhausting a limited search capacity; otherwise, she abstains. The model is built from three primitives: a preference, a search capacity, and a threshold. I study this in two versions. In a deterministic version, the search capacity and the threshold are fixed, which lets me fix ideas and expose a sharp identification problem: knowing the decision maker's choice from every possible list is not enough to uniquely recover her primitives, even though, as I show, the class of choice functions consistent with the procedure is itself fully pinned down by two simple axioms. This motivates the paper's main model, in which both the search capacity and the threshold are random, so that observed choice is stochastic even though the underlying search procedure, conditional on the primitives, is not.

The paper's main results show that randomizing the primitives does more than add noise: it resolves the deterministic model's identification failure and delivers a model with sharp testable content. I show that (i) despite the deterministic model's partial identification of primitives, the choice functions it generates admit a complete axiomatic characterization; (ii) the underlying preference is always identified from stochastic choice, even though it need not be in the deterministic model; (iii) the joint distribution over search capacity and threshold can be partially identified directly from choice frequencies, and I characterize precisely which of its values can and cannot be recovered this way; (iv) under an independence condition between search capacity and threshold, this joint distribution is fully identified; and (v) the model is fully characterized by six testable axioms on stochastic choice from lists: I show that they are both necessary and sufficient.

2 Related Literature

This combination of a random search depth and a random threshold, applied to an environment in which only the order of a list, and not the underlying menu, varies, is, to the best of my knowledge, new to the literature. The model I propose builds on Ishii et al. (2021), a paper that analyzes a stochastic choice environment over lists in which the only variation is the order in which alternatives are presented, with no menu variation whatsoever. A key ingredient of the Ishii et al. (2021) model is that the decision maker has a limited search-depth primitive which determines the number of alternatives she considers before choosing; the authors show that this Random Depth Model, together with two special cases of it, is fully characterized by choice from lists alone. The model presented in this paper takes this random-depth idea and combines it with an additional mechanism: I assume that the decision maker chooses the first alternative presented to her that satisfies a given threshold, rather than the best alternative she has observed according to a fixed preference. If none of the alternatives presented to the decision maker satisfy the threshold she has in mind, she abstains from choosing.

There are several papers in the literature that study a decision maker's behavior whenever alternatives are presented in varying lists. One example is Rubinstein and Salant (2006), which axiomatizes decisions made from a list whenever the decision maker faces both list and menu variation. The authors present two axioms that are the list-version of typical axioms in the choice literature: partition independence (dividing a list into several sublists, and choosing from them, leads to the same result as choosing from the original list) and list independence of irrelevant alternatives (omitting unchosen elements from a list does not alter choice). The model presented in this paper differs from Rubinstein and Salant (2006) in that I do not consider menu variation, so the idea of dividing a list and eliminating unchosen alternatives does not apply here.

Regarding the notion of satisficing a threshold, Kovach and Ülkü (2020) present and fully axiomatize a stochastic choice model that features random thresholds and the possibility of choice abstention. Given a menu, their decision maker draws a random threshold and then searches the menu, using a fixed underlying list order, for the first alternative that is at least as good as the threshold; the authors show that this model is a special Random Utility Model, and that it is testable using two simple axioms. Their analysis focuses on behavior when the decision maker faces menu variation with an implicit, fixed list order. The model in this paper borrows their central idea, that the decision maker chooses the first alternative in the (implicit or explicit) list that surpasses a random threshold, but places it in an environment where it is the list, rather than the menu, that varies, and where the decision maker's search depth is itself random rather than unbounded. Regarding thresholds, Manzini et al. (2024) also study a model of sequential, threshold-based behavior over lists; however, the authors focus their analysis on approval decisions (e.g., favoriting, sharing, or wish-listing an item), rather than on a single, final choice, and their continuation probabilities may depend on the entire history of predecessors rather than on a fixed search-depth primitive.

More broadly, the idea that a decision maker searches sequentially and stops once she is satisfied connects this paper to a literature on search and satisficing that goes beyond list environments. Caplin et al. (2011) provide direct experimental evidence for sequential search and a satisficing stopping rule, using a choice task in which both the search process and the final choice are observed. Aguiar et al. (2016) axiomatize a model in which a decision maker with a fixed preference and a fixed reservation utility searches a menu in an order that varies randomly across choice occasions, and characterize exactly when stochastic choice data are consistent with this behavior. Like Kovach and Ülkü (2020), both papers study satisficing under menu variation; the environment in this paper instead holds the menu fixed throughout and randomizes only the order in which that single menu is presented, together with the decision maker's search depth.

The contribution of this paper is best understood not merely as filling a previously unstudied combination, but as the specific implications that emerge from combining these two ingredients, random search depth and a random threshold, within a list environment. Ishii et al. (2021) study list variation with a random search depth, but no threshold, so search always ends at the best alternative the decision maker has observed. Kovach and Ülkü (2020), Caplin et al. (2011), and Aguiar et al. (2016) study random thresholds and satisficing behavior, but under menu variation, with search either unbounded or governed by an implicit, fixed list order rather than a random depth. A random search depth on its own would leave the threshold, and with it the preference-identification problem the deterministic model exposes, entirely unaddressed; a random threshold on its own, without a bound on search, would leave abstention unconstrained by list position. It is precisely the interaction of the two, within an environment where only the list varies, that produces both the deterministic model's identification failure and its resolution once the threshold is randomized, together with the sharp identification and characterization results of Section 5. To the best of my knowledge, no existing paper studies this combination, and I close the resulting gap with a full if-and-only-if characterization.

The remainder of the paper is organized as follows. Section 3 introduces the basic ingredients and notation used throughout the paper. Section 4 presents the deterministic version of the model as a benchmark, and shows why its identification failure motivates randomizing the primitives. Section 5 presents the paper's main model, together with its identification results and a proposed axiomatization, completing an if-and-only-if characterization of choice from lists under this model. Section 6 discusses an extension of the model, and Section 7 concludes with a discussion of directions for future work.

3 Model Setup

I now set up the environment formally. Let X denote the set of alternatives, which I assume is finite, with $|X| = n$. Since the key feature I want the model to capture is that choice depends on the order in which alternatives are presented, rather than on which alternatives are available, I consider an environment in which the decision maker always faces the entire set X, arranged in an order that varies: choice is made from lists.

Definition 1. A **list** L is any linear order on X.[1]

Given a list, I interpret xLy as x precedes y. A list can be written as $x_1Lx_2Lx_3Lx_4L\dots$, or simply $x_1x_2x_3x_4\dots$. I define $L(x)$ as the position of element x in list L, and I write $L^{-1}(j)$ for its inverse, the alternative placed in position j of L (so $L^{-1}(L(x)) = x$ for every $x \in X$). Let $\mathcal{L}$ be the set of all lists over X.

In addition, I want to allow for the possibility that none of the alternatives that the decision maker has considered satisfies the threshold she has in mind. Hence, another characteristic that I want the model to feature is the possibility of choice abstention. In this sense, I consider an object $a^\star \notin X$ that represents the act of abstaining from choosing an alternative in X.

1 A linear order is any binary relation that is complete (xLy or yLx for every $x, y \in X$), transitive (xLy, yLz implies that xLz for every $x, y, z \in X$), and anti-symmetric (xLy and yLx implies $x = y$).

Definition 2. Given a list L and a subset S of X, $\min(S, L)$ denotes the first placed member of S according to list L. That is, $\min(S, L) = x$ if and only if $x \in S$ and $L(x) \leq L(y)$ for every $y \in S$. If $S = \emptyset$ then I set $\min(S, L) = a^\star$.

Definition 3. A (deterministic) **choice function** is a map $c : \mathcal{L} \to X \cup \{a^\star\}$. I interpret $c(L)$ as the choice made by the decision maker whenever she faces list L.

Definition 4. A **stochastic choice function** is a map $Q : X \cup \{a^\star\} \times \mathcal{L} \to [0, 1]$ satisfying:

$$\sum_{x \in X \cup \{a^\star\}} Q(x, L) = 1,$$

for every list L.

Both definitions presume that c or Q is observed on the entire domain $\mathcal{L}$ of lists, i.e., on every permutation of X. This is a standard, and for a theoretical characterization a necessary, richness assumption: the axioms of Sections 4 and 5 are statements about behavior across lists that differ only in ways the axioms themselves specify, and testing them presupposes that choice from each such list is available for comparison. An empirical application in which only some orderings of X are ever observed, as is typical of real ranking data, calls for a separate identification and testing strategy suited to that more limited domain; the results below characterize what full list variation reveals in principle; they are the natural benchmark against which such incomplete-domain strategies should be judged, but do not by themselves deliver one.

I study the decision maker's search-and-threshold procedure in two steps. Section 4 first fixes the search capacity and threshold as known, deterministic primitives. This is a useful benchmark: it isolates the mechanics of the search procedure from any randomness, and it exposes a specific identification failure that motivates the rest of the paper. Section 5 then randomizes both primitives, which is the model I take to the data of choice frequencies and the paper's central object of study.

4 The Deterministic Model

As a benchmark, I first consider the case in which the decision maker's search capacity and threshold are fixed and commonly known, so that behavior is deterministic. Given a list L, the decision maker observes a fixed number of alternatives and chooses the first one she has considered that satisfies a fixed threshold she has in mind.

Definition 5. A choice function c is a **Depth-Threshold Model** (DTM) if there exist $\succeq$, a linear order on X, a depth parameter $k \in \{1, \ldots, n\}$, and a fixed threshold $t \in X$ such that for every list L:

$$c(L) = \min\left(A(\succeq, L, k, t), L\right), \tag{1}$$

where $A(\succeq, L, k, t) = \{x \in X : x \succeq t \text{ and } L(x) \leq k\}$. If c is a DTM, I write $c = c^{\succeq, k, t}$.

4.1 Example

Let $X = \{x, y, z\}$ and let $\succeq$ be such that $x \succ y \succ z$.[2] In this set-up, there are only six possible lists that present the different alternatives in X to the decision maker. Assuming that the decision maker has a depth capacity equal to two and a threshold of y ($k = 2, t = y$), the choices

2 By $\succ$, I mean the strict part of $\succeq$. In other words, $x \succ y$ if and only if $x \succeq y$ and not $y \succeq x$.

made by the decision maker are displayed in Table I.

Table I: Deterministic Model Induced Choice Example

List	xyz	xzy	yxz	yzx	zxy	zyx
$c^{\succeq,k,t}(L)$	x	x	y	y	x	y

For example, considering $L = xyz$, since the decision maker has a depth capacity of $k = 2$ she will only observe $\{x, y\}$. However, x is the first element in the list to satisfy $x \succeq t = y$, hence it is chosen. In list $L' = zyx$, the decision maker only observes $\{y, z\}$, and y is the first element in the list to surpass the threshold.

If instead I consider a depth capacity of $k = 2$ and $t = x$, the choices made by the decision maker are displayed in Table II.

Table II: Deterministic Model Induced Choice Example

List	xyz	xzy	yxz	yzx	zxy	zyx
$c^{\succeq,k,t}(L)$	x	x	x	$a^\star$	x	$a^\star$

Consider list $L = yzx$. If this list is observed by the decision maker, she will only consider $\{y, z\}$. However, neither of these alternatives satisfies the threshold restriction. Hence the set $A(\succeq, L, k, t)$ is empty, which implies that the decision maker abstains.

4.2 Identification of Primitives

I first note that identification in the DTM is incomplete, i.e., given an observed $c^{\succeq,k,t}$ there could exist more than one triple $(\succeq, k, t)$ generating the same choice function. To see this, consider the triples $(x \succ y \succ z, k = 2, t = z)$ and $(x \succ y \succ z, k = 3, t = z)$. In both cases, given a list L, the decision maker chooses the first element she sees. Hence, I am only able to partially identify the primitives of the DTM.

The following proposition allows me to (partially) identify the depth capacity primitive k.

Proposition 1. *If $c^{\succeq,k,t}(L) = w$ with $w \in X$ then $k \geq L(w)$.*

This proposition states that if the decision maker's choice is w, and this element is in position $L(w)$, then the depth capacity must be at least $L(w)$; otherwise, the decision maker would not have considered w.

In order to identify $\succeq$, note that in the DTM, if the decision maker chooses w, then every alternative x that precedes it must not satisfy $x \succ w$. Hence, I use the following proposition to identify $\succeq$.

Proposition 2. *If there exists a list L such that $c^{\succeq,k,t}(L) = x$ and $L(y) \leq L(x)$ then $x \succeq y$.*

The proof of Proposition 2 can be found in Appendix A. Despite this partial identification, Section 4.3 shows that the class of choice functions generated by some DTM is nonetheless fully characterized.

4.3 Axiomatization

Any DTM satisfies the following monotonicity property.

Proposition 3. *If L, L' are two lists such that $x = c(L)$ and $\{y : L'(y) \leq L'(x)\} \subseteq \{y : L(y) \leq L(x)\}$, then $x = c(L')$.*

This property requires that if x is chosen from list L, then x must also be chosen from any other list L' in which the set of alternatives placed at or before x's position is no larger than in L. Intuitively, if x managed to be chosen despite having to precede a certain set of alternatives, then facing a smaller version of that same set of predecessors should not prevent x from being chosen again. The proof of Proposition 3 can be found in Appendix A.

Proposition 3 is necessary but not sufficient for a choice function to be a DTM. To see this, consider the choice function over a set $X = \{x, y, z\}$ displayed in Table III.

Table III: Example

List	xyz	xzy	yxz	yzx	zxy	zyx
$c(L)$	x	x	y	y	$a^\star$	y

This choice function clearly satisfies Proposition 3. However, there is no triple $(\succeq, k, t)$ that renders these choices a DTM. Suppose otherwise. Then $c(zyx) = y$ implies that $k \geq 2$ and $y \succeq z$. The threshold t cannot be $t = x$, otherwise $c(zyx) \neq y$ (it would be either x or abstention). The threshold cannot be $t = y$, otherwise $c(zxy) = x$. Hence, the threshold must be $t = z$. However, this would imply $c(zxy) = z$, which is not the same as abstention.

Proposition 3 fails to rule this choice function out because it only restricts behavior *conditional on* some alternative already being chosen; it says nothing about which alternatives can be chosen at all, nor about how abstention across different lists must relate to one another. This section presents two axioms that close exactly these two gaps and, together, fully characterize the DTM. Throughout, say that $y \in X$ is **never chosen** under c if $c(L) \neq y$ for every list $L \in \mathcal{L}$, and let $N(c) := \{y \in X : y \text{ is never chosen under } c\}$.

Axiom 1 (Domination)**.** If $x = c(L)$ for some list L, and $L(y) < L(x)$, then $y \in N(c)$.

This axiom requires that every alternative y that ever precedes an actual choice must itself be never chosen, under any list whatsoever. Intuitively, if the decision maker was willing to pass over y on her way to choosing x, then y must have failed to satisfy her threshold; since satisfying the threshold does not depend on the list being faced, y fails it everywhere, so y can never be chosen, no matter how the rest of the list is arranged or where y is placed within it.

Proposition 4. *Any DTM satisfies Axiom 1.*

The proof of Proposition 4 can be found in Appendix A.

Axiom 1 already does more work than Proposition 3: it identifies $N(c)$, the set of alternatives that never satisfy the threshold, directly from observed choice, without reference to $\succeq$, k, or t. What remains is to discipline abstention itself, so that which lists lead the decision maker to abstain is governed by a single, common search depth rather than varying erratically from list to list. For a list L, let

$$\beta(L) := \max\{j \in \{0, 1, \ldots, n\} : \{L^{-1}(1), \ldots, L^{-1}(j)\} \subseteq N(c)\}$$

denote the length of the initial run of never-chosen alternatives in L (with $\beta(L) = 0$ if $L^{-1}(1) \notin N(c)$). In words, $\beta(L)$ counts how many alternatives at the start of L the decision maker would have to pass over before she could possibly stop, regardless of how deep her search goes: since every one of these first $\beta(L)$ alternatives is never chosen under any list, none of them can ever satisfy her threshold, so a search that only reaches them necessarily ends in abstention. A list with a large $\beta(L)$ is therefore one that opens with a long stretch of alternatives no search depth could ever make her choose.

Axiom 2 (Threshold Consistency)**.** If L, L' are two lists such that $c(L) = a^\star$ and $c(L') \neq a^\star$, then $\beta(L') < \beta(L)$.

This axiom requires that the initial run of never-chosen alternatives be strictly shorter in any list where the decision maker actually chooses something than in any list where she abstains instead. Intuitively, abstention occurs precisely when the decision maker exhausts her search depth without ever observing an alternative that clears her threshold, so the never-chosen run she sits through, unrewarded, before giving up must always be longer than the one she sits through before reaching an alternative she is willing to take; a single depth parameter can then be placed strictly between the two.

Proposition 5. *Any DTM satisfies Axiom 2.*

The proof of Proposition 5 can be found in Appendix A.

It is worth revisiting Table III in light of Axiom 2. There, $N(c) = \{z\}$: the abstaining list zxy has $\beta(zxy) = 1$, and the non-abstaining list zyx, which chooses y, has $\beta(zyx) = 1$ as well. Since $1 \nless 1$, this choice function violates Axiom 2, which is exactly why no DTM can generate it: the hand argument above, that no threshold t can rationalize both $c(zxy) = a^\star$ and $c(zyx) = y$, is in effect a proof that no single search depth k can separate this list's abstention from that list's choice.

Axiom 1 and Axiom 2 are jointly sufficient for the DTM, together with the mild requirement that the decision maker does not abstain from every list. This requirement rules out only the degenerate choice function $c \equiv a^\star$, which no DTM can generate in the first place: the decision maker's most preferred alternative, whenever placed first in the list, always satisfies every possible threshold and is observed regardless of the realized search depth, so it is always chosen (an argument that parallels Proposition 6 in Section 5).

Theorem 1. *A choice function c with $c(L) \neq a^\star$ for at least one list L is a DTM if and only if it satisfies Axiom 1 and Axiom 2.*

Necessity follows from Proposition 4 and Proposition 5. For sufficiency, the proof constructs $\succeq$, k, and t directly from c. Axiom 1 identifies $N(c)$ from the data alone, which pins down exactly which alternatives the decision maker's threshold rules out; any linear order that ranks every alternative in $X \setminus N(c)$ above every alternative in $N(c)$ can then serve as $\succeq$ (the ranking within each of these two sets plays no role in generating c, which is exactly why Section 4.2 shows it cannot be recovered from choice), and t is set to the $\succeq$-worst alternative in $X \setminus N(c)$. The depth k is set equal to the shortest never-chosen run $\beta(L)$ across all lists L at which the decision maker abstains, or to n if she never abstains. Axiom 2 guarantees that $\beta(L') < k \leq \beta(L)$ for every non-abstaining list L' and every abstaining list L — strict on the non-abstaining side, and, by construction, tight with equality for a list attaining the minimum that defines k — while Axiom 1 guarantees that whenever $c(L) \neq a^\star$, $c(L)$ is exactly the first alternative of L lying outside $N(c)$. Together, these two facts show that the resulting DTM reproduces c exactly. The full proof can be found in Appendix A.

5 The Stochastic Model

I now present the paper's main model, in which both the search depth and the threshold are randomized. Let $\sigma : \{1, \dots, n\} \times X \to [0, 1]$ denote a joint probability density over $\{1, \dots, n\} \times X$, i.e. $\sigma(k, t) \geq 0$ for every (k, t) and $\sum_{k,t} \sigma(k, t) = 1$, and suppose in addition that $\eta(t) := \sum_{k=1}^{n} \sigma(k, t) > 0$ for every $t \in X$: every alternative has some chance, at some search depth, of being the decision maker's threshold, though any single depth-threshold pair may itself have zero probability. Therefore $\sigma(k, t)$ is the probability that the decision maker observes the first k elements of a list having in mind a threshold of t. I now present the functional form assumed for stochastic behavior.

Definition 6. A stochastic choice function $Q : X \cup \{a^\star\} \times \mathcal{L} \to [0, 1]$ is a **Random Depth Random Threshold Model** (RDRTM) if for every $x \in X$:

$$Q(x, L) = \sum_{(k,t): x = c^{\succsim, k, t}(L)} \sigma(k, t). \tag{2}$$

For $a^\star$, I define:

$$Q(a^\star, L) = 1 - \sum_{x \in X} Q(x, L), \tag{3}$$

which captures the probability that the decision maker abstains from choosing when considering list L.

Equation (2) says that the probability of choosing x from list L is the total probability mass placed on those pairs (k, t) of search depth and threshold that would lead a decision maker following the deterministic DTM procedure, with preference $\succsim$, to choose x from L: I simply sum $\sigma(k, t)$ over every realization of the primitives consistent with choosing x. In this model, the decision maker chooses x in list L if it satisfies being in one of the first k positions of the list, and it is the **first** alternative in L that surpasses the threshold t.

5.1 Example

Suppose that $X = \{x, y, z\}$ is the set of alternatives, with preferences over X given by $x \succ y \succ z$. Given this preference, the probability that each element is chosen according to the model is displayed in Table IV.[3]

Table IV: Stochastic Model Induced Choice Example

List	$Q(x, L)$	$Q(y, L)$	$Q(z, L)$
xyz	1	0	0
xzy	1	0	0
yxz	$\sigma(2, x) + \sigma(3, x)$	$\sum_{k=1}^{3} \sigma(k, y) + \sigma(k, z)$	0
yzx	$\sigma(3, x)$	$\sum_{k=1}^{3} \sigma(k, y) + \sigma(k, z)$	0
zxy	$\sum_{k=2}^{3} \sigma(k, x) + \sigma(k, y)$	0	$\sum_{k=1}^{3} \sigma(k, z)$
zyx	$\sigma(3, x)$	$\sigma(2, y) + \sigma(3, y)$	$\sum_{k=1}^{3} \sigma(k, z)$

3 The probability of abstention in a given list L is $Q(a^\star, L) = 1 - Q(x, L) - Q(y, L) - Q(z, L)$, which is excluded to economize space.

For example, consider the list yxz. In order to choose x, the decision maker must observe at least two elements of the list. However, if the threshold she has in mind is either $t = y$ or $t = z$, then there is an element in this list (y) that comes before x and satisfies $y \succeq t$; she would then choose y instead of x. In this sense, the only possibility for a decision maker to choose x is if she observes two or three alternatives, having $t = x$ as her threshold. Therefore, $Q(x, yxz) = \sigma(2, x) + \sigma(3, x)$.

In list zyx, since z is the first element on the list, a decision maker chooses z only if the threshold she has in mind is $t = z$: otherwise, if $t = x$ or $t = y$, she would not pick the first element of the list. However, if she observes two or three elements of the list with $t = z$ in mind, she will still choose z, since it is the first element on the list to satisfy the threshold. Therefore, $Q(z, zyx) = \sum_{k=1}^{3} \sigma(k, z)$.

This example illustrates some of the properties satisfied by this choice model, which I discuss below.

5.2 Properties of the Model

The following propositions capture some of the key aspects of the behavior induced by the RDRTM.

Proposition 6. *Let $x^\star \in X$ be such that $x^\star \succ x$ for every $x \in X \setminus \{x^\star\}$. Then, if L is such that $L(x^\star) = 1$, $Q(x^\star, L) = 1$.*

This proposition states that the most preferred alternative in X, whenever it happens to be placed first in the list, is chosen with certainty. This is because $x^\star$ satisfies every possible threshold $t \in X$ (since $x^\star \succeq t$ for any t), and, being first in the list, it is always observed regardless of the realized search depth $k \geq 1$; hence no realization of (k, t) can lead the decision maker to either pass over $x^\star$ or abstain. The proof of Proposition 6 can be found in Appendix B. The following property highlights the importance of order in this model: if in a given list L two elements are such that the preferred one comes first, the latter is chosen with probability zero.

Proposition 7. *If $x \succ y$ and L is a list such that $L(x) < L(y)$ then $Q(y, L) = 0$.*

Intuitively, whenever a better alternative x precedes a worse alternative y in the list, any threshold that y would satisfy is also satisfied by x; since the decision maker searches in order, she would then stop at x before ever reaching y, regardless of her search depth or threshold, so y is never chosen. The proof of Proposition 7 can be found in Appendix B. The following proposition characterizes the decision process leading a decision maker who follows an RDRTM to choose an alternative $x \in X$.

Proposition 8. *Let Q be an RDRTM. Given an alternative $x \in X$ and a list L:*

$$Q(x, L) = \sum_{k=L(x)}^{n} \sum_{t \in \Gamma(\succeq, L, x, k)} \sigma(k, t), \tag{4}$$

where $\Gamma(\succeq, L, x, k) = \{t : x \succeq t \text{ and } t \succ z \text{ if } L(z) < L(x) \leq k\}$.

This proposition states that, in order for a decision maker to choose x in list L, first x must satisfy some threshold t ($x \succeq t$). Also, every element z that precedes x in L ($L(z) < L(x)$) must not satisfy $z \succeq t$; otherwise, the decision maker would choose z instead of x.

A closer look at $\Gamma(\succeq, L, x, k)$ reveals that it does not actually depend on k, and that it depends on L only through the position of x and the identity of its single best predecessor. For a list L and $x \in X$ with $\{y : L(y) < L(x)\} \neq \emptyset$, let $z^\star(x, L)$ denote the $\succeq$-best element of $\{y : L(y) < L(x)\}$, i.e. the most preferred alternative that precedes x in L. Also let

$$\tau(m, t) := \sum_{k=m}^{n} \sigma(k, t)$$

denote the probability that the decision maker's search depth reaches at least m and her threshold is t.

Proposition 9. *Let Q be an RDRTM. For every $x \in X$ and every list L, $\Gamma(\succeq, L, x, k)$ is the same set for every $k \geq L(x)$, and*

$$Q(x, L) = \sum_{t\,:\,z^\star(x,L) \prec t \preceq x} \tau(L(x), t), \tag{5}$$

with the sum ranging over $\{t : t \preceq x\}$ if $\{y : L(y) < L(x)\} = \emptyset$.

The proof of Proposition 9 can be found in Appendix B. Equation (5) says that only the single best predecessor of x in L, not the rest of the predecessor set, determines x's choice probability, and that this probability depends on x's position $L(x)$ in L only through the depth-truncated mass $\tau(L(x), \cdot)$. Proposition 8 has the following further corollary.

Proposition 10. *Let Q be an RDRTM, L, L' two lists, and $x \in X$. If $\{y : L'(y) \leq L'(x)\} \subseteq \{y : L(y) \leq L(x)\}$ then $Q(x, L') \geq Q(x, L)$.*

This proposition is the stochastic analog of the deterministic model's Proposition 3: shrinking the set of alternatives that precede x in a list can only weakly increase x's probability of being chosen, since x then faces weakly fewer potential predecessors that could satisfy the decision maker's threshold and be chosen instead of x, for every realization of the search depth and threshold. The proof of Proposition 10 can be found in Appendix B. Note that if $\{y : L'(y) \leq L'(x)\} = \{y : L(y) \leq L(x)\}$ then Proposition 10 implies $Q(x, L) = Q(x, L')$: garbling the predecessors of x in a list leaves its choice probability intact.

5.3 Identification of Primitives

This section discusses how to identify the two primitives that determine behavior in this model: $\succeq$ and σ. Throughout, I only assume that I observe $Q(x, L)$ for every $\{x, L\}$ and that $\succeq$ is a linear order.

Proposition 11. *Consider two lists L and L' such that $L = x \ldots$ and $L' = y \ldots$ with $x \neq y$. If $Q(x, L) > Q(y, L')$ then $x \succ y$.*

The proof of Proposition 11 can be found in Appendix B. The intuition behind this identification technique is simple: given that the decision maker chooses the first element satisfying a threshold, comparing x, y by placing each in the first position of a list, $Q(x, L) > Q(y, L')$ means that there are thresholds that x satisfies but y does not; otherwise, given that it occupies the first position, y would be chosen. Hence x is preferred to y. Compared with the deterministic model, in the stochastic version I can always identify $\succeq$.

Head-of-list probabilities identify more than just $\succeq$: they pin down the entire marginal distribution of thresholds as well. By Proposition 9, for any list L with $L(x) = 1$, $Q(x, L)$ depends only on x; write $Q_1(x)$ for this common value (formally introduced as such in Section 5.4). Ordering X from best to worst as $x_1 \succ x_2 \succ \cdots \succ x_n$, the following proposition shows that differencing Q_1 along this order recovers η directly, with no reference to search depth and no use of Definition 7 below.

Proposition 12. *Let Q be an RDRTM with underlying preference $x_1 \succ x_2 \succ \cdots \succ x_n$. Then*

$$\eta(x_i) = Q_1(x_i) - Q_1(x_{i+1}) \text{ for } i < n, \qquad \eta(x_n) = Q_1(x_n). \tag{6}$$

The proof of Proposition 12 can be found in Appendix B. Intuitively, $Q_1(x_i)$ is the probability that the decision maker's threshold is weakly worse than x_i, so moving from x_{i+1} to its immediate senior x_i adds exactly the probability mass at x_i itself to this cumulative sum. Together with Proposition 11, this shows that stochastic choice from lists always identifies both the underlying preference and the complete threshold marginal η; only the finer joint object $\sigma(k,t)$, to which I turn next, requires more work, and is not always fully identified.

Once $\succeq$ is identified, the following proposition allows me to identify $\sigma(k,t)$.

Proposition 13. *Consider two alternatives x and y such that $x \succ y$ and there exists no w such that $x \succ w \succ y$. Consider two lists L, L' such that $L(x) = m, L(y) = m+1, L'(x) = m+1, L'(y) = m$, and $L(z) = L'(z)$ for all $z \in X \setminus \{x,y\}$ (i.e., L, L' only swap the positions of x and y). Suppose that in both L, L' every alternative preceding x is worse than it. Then:*

$$\sigma(m,x) = Q(a^\star, L') - Q(a^\star, L).$$

This proposition provides a way to recover $\sigma(m,x)$ directly from observed abstention probabilities. Swapping x and y, adjacent in preference, across positions m and $m+1$ leaves the predecessor set of every other alternative unchanged, so it can only affect the probability of abstention through the fate of the pair $(k,t) = (m,x)$: whenever the decision maker draws exactly this depth and threshold, she chooses x in list L (since x is now observed, in position m, and clears her own threshold), but she can no longer reach x in list L' (where x has been pushed to position $m+1$) and, since y does not satisfy a threshold of x, she abstains instead. Hence the entire difference in abstention probability between L' and L is due to this single realization, giving $\sigma(m,x)$. The proof of Proposition 13 can be found in Appendix B. This strategy identifies the joint probability that the decision maker's threshold is x and her search depth is m, not the (list-dependent) probability of choosing x at depth m: by Proposition 8, the latter is $\sum_{t \in \Gamma(\succeq, L, x, m)} \sigma(m,t)$, generally a sum over several threshold values rather than the single cell $\sigma(m,x)$. However, there is a caveat: if for a given $z \in X$ there is no $\{y, L\}$ such that $z \succ y$ and $m = L(z) < L(y)$, then $\sigma(m,z)$ **cannot** be identified with this procedure.

Proposition 14. *Consider an RDRTM Q. Then identification using the procedure of Proposition 13 will always be incomplete: for every RDRTM there exists an alternative $z \in X$ for which there is no $\{y, L\}$ such that $z \succ y$ and $m = L(z) < L(y)$.*

The proof of Proposition 14 can be found in Appendix B.

An additional assumption allows me to uniquely identify $\sigma(m,x)$ for every (m,x).

Definition 7. The density σ satisfies **Independence** if there exist two probability densities μ over $\{1,\dots,n\}$ and η over X such that for every (m,x):

$$\sigma(m,x) = \mu(m)\eta(x).$$

In order to clarify how Independence allows me to use the procedure of Proposition 13 to uniquely identify σ, suppose that $X = \{x_1, \ldots, x_p\}$ and that $x_1 \succ x_2 \succ \cdots \succ x_p$. Using the procedure of Proposition 13, I am able to identify the following joint probabilities:

$$\begin{pmatrix} \sigma(1,x_1) & \sigma(1,x_2) & \ldots & \sigma(1,x_{p-2}) & \sigma(1,x_{p-1}) & U \\ \sigma(2,x_1) & \sigma(2,x_2) & \ldots & \sigma(2,x_{p-2}) & U & U \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma(n-2,x_1) & \sigma(n-2,x_2) & U & \ldots & U & U \\ \sigma(n-1,x_1) & U & U & \ldots & U & U \end{pmatrix}$$

In this matrix, U stands for unidentified. For example, considering $k = 1$, the procedure of Proposition 13 identifies $\sigma(1, x_m)$ for $m = 1, \ldots, p-1$, since I can always swap x_m with x_{m+1} considering a list L such that $L(x_m) = 1$. However, this swapping process cannot be carried out with x_p.

In order to identify $\sigma(2, x_m)$, I can use the procedure of Proposition 13 for $m = 1, \ldots, p-2$. I cannot identify, for example, $\sigma(2, x_{p-1})$, since doing so requires a list L such that $L(x_{p-1}) = 2, L(x_p) = 3$, and the difference between the abstention probability in L and in the list L' that only swaps these two elements. However, in any such list L, the element x with $L(x) = 1$ always dominates x_{p-1} ($x \succ x_{p-1}$), since $\{x_{p-1}, x_p\}$ are the least preferred alternatives in X. This implies that for every L with $L(x_{p-1}) = 2, L(x_p) = 3$, $Q(x_{p-1}, L) = 0$, leading to an identification problem.

By Proposition 12, η is already fully identified directly from head-of-list probabilities, with no need for Independence or for the swap procedure above. Independence therefore only remains to deliver μ, and hence all of σ, which it does using nothing more than the best alternative x_1:

1. Using the procedure of Proposition 13, identify $\sigma(k, x_1)$ for $k = 1, \ldots, n-1$ (recall that $x_1 \succ x$ for all $x \in X \setminus \{x_1\}$).
2. Consider a list L with $L(x_1) = n$. Since $x_1 \succeq t$ for every $t \in X$, the decision maker chooses x_1 from L only if her threshold is $t = x_1$ itself, since any other threshold is already satisfied by some predecessor of x_1, and only if her search depth reaches n. Hence $Q(x_1, L) = \sigma(n, x_1)$, identifying the one remaining cell $\sigma(n, x_1)$ left unidentified by Step 1.
3. Since $\eta(x_1) = \sum_{k=1}^{n} \sigma(k, x_1) > 0$ is already identified and strictly positive by Proposition 12, Independence gives
$$\mu(k) = \frac{\sigma(k, x_1)}{\eta(x_1)}, \quad k = 1, \ldots, n.$$
4. With both μ and η now identified, Independence gives $\sigma(k,t) = \mu(k)\eta(t)$ for every (k,t).

This route divides only by $\eta(x_1)$, which Section 5 assumes throughout to be strictly positive; unlike dividing by a specific depth mass such as $\mu(1)$, no further positivity assumption is needed, since Independence itself places no restriction on which depths occur with positive probability.

To make this identification procedure concrete, return to the example of Table IV, where $x \succ y \succ z$. Proposition 12 recovers the entire threshold marginal directly from the table's head-of-list entries: $\eta(z) = Q_1(z) = \sum_{k=1}^{3} \sigma(k,z)$ (read off list zxy or zyx), $\eta(y) = Q_1(y) - Q_1(z)$ (read off list yxz or yzx), and $\eta(x) = 1 - Q_1(y)$, using $Q_1(x) = 1$ by Proposition 6. Proposition 13 then recovers individual depth-threshold cells: swapping x and y across positions 1 and 2, from $L = xyz$ to $L' = yxz$, gives

$$\sigma(1, x) = Q(a^\star, yxz) - Q(a^\star, xyz) = Q(a^\star, yxz),$$

since x heads list $L = xyz$ and is therefore chosen with certainty, leaving $Q(a^\star, xyz) = 0$. By Proposition 14, this same swap procedure cannot recover $\sigma(m, z)$ for any m: z is $\succeq$-worst, so it can never serve as the better, undominated element of an adjacent swap pair, and these three cells are only recovered once Independence is imposed, following the steps above.

5.4 Model's Characterization

This section presents six axioms on stochastic choice from lists and shows that they are not only necessary, but also sufficient.

Axiom 3 (Head Regularity)**.** For every $x \in X$ and every two lists L, L' with $L(x) = L'(x) = 1$: $Q(x, L) = Q(x, L') =: Q_1(x)$, with $Q_1(x) > 0$; and $Q_1(x) \neq Q_1(y)$ for every $x \neq y$.

This axiom requires three things about the probability of being chosen when heading a list. First, this probability must not depend on how the remaining alternatives are arranged behind the leader, a minimal consistency requirement, since nothing behind x can matter to her until the decision maker has already passed over x itself. Second, it must be strictly positive for every alternative: since every alternative has some chance, however small, of being the decision maker's threshold, even the worst alternative, if placed first, has some chance of being chosen outright. Third, no two alternatives can tie in this probability, since a strictly better alternative is always strictly more likely to satisfy whatever threshold the decision maker draws. Axiom 3 therefore reveals the underlying preference directly: define $x \succ y$ if and only if $Q_1(x) > Q_1(y)$; since Q_1 is real-valued and, by the axiom, injective on the finite set X, this is automatically a linear order, with no separate argument needed to establish completeness, transitivity, or antisymmetry.

Proposition 15. *Any RDRTM satisfies Axiom 3.*

The proof of Proposition 15 can be found in Appendix B.

Definition 8. Let $\succ$ denote the binary relation on X given by $x \succ y$ if and only if $Q_1(x) > Q_1(y)$, well defined and a linear order on X by Axiom 3. Let $\succeq$ denote its weak counterpart: $x \succeq y$ if and only if $x = y$ or $x \succ y$. For $z \in X$, let $\rho(z) := \#\{w \in X : z \succ w\}$ denote the number of alternatives strictly worse than z under $\succ$. Whenever $\rho(z) \geq 1$, let z^- denote z's $\succ$-immediate junior: the unique $w \in X$ such that $z \succ w$ and there is no $v \in X$ with $z \succ v \succ w$.

The number $\rho(z)$ equals 0 exactly when z is $\succ$-worst, and otherwise counts how many alternatives z can still afford to have precede it before every remaining candidate outranks it. The junior z^- is the single alternative directly below z in this order: the natural, harmless predecessor to place immediately in front of z without affecting whether she is chosen. Order X from best to worst as $x_1 \succ x_2 \succ \cdots \succ x_n$, and let $\tau(1, x_i) := Q_1(x_i) - Q_1(x_{i+1})$ for $i < n$ and $\tau(1, x_n) := Q_1(x_n)$; telescoping, $\sum_{i=1}^{n} \tau(1, x_i) = Q_1(x_1)$.

Axiom 4 (List Independence)**.** For every $x \in X$ and every two lists L, L' with $L(x) = L'(x) = m \geq 2$: if $\{y : L(y) < L(x)\}$ and $\{y : L'(y) < L'(x)\}$ have the same $\succ$-best element, then $Q(x, L) = Q(x, L')$.

This axiom says that x's probability of being chosen depends on its predecessors only through the single best one among them, not on the rest of the set: since the decision maker's search stops at the first alternative to clear her threshold, only the best-placed obstacle she has already passed over could have stopped her instead of x, so it alone determines whether x is still reachable. Any RDRTM satisfies this axiom: it is exactly Proposition 9.

Let $\perp$ denote the absence of a predecessor: a list L has best predecessor $\perp$ for x if $L(x) = 1$, i.e. no alternative precedes x in L.

Definition 9 (Canonical Choice Probability)**.** For $x \in X$, a position $m \in \{1, \ldots, n\}$, and $z \in X \cup \{\perp\}$ such that some list L has $L(x) = m$ and best predecessor z, let $\bar{Q}(x; m, z)$ denote the common value of $Q(x, L)$ across every such list, well defined by Axiom 3 (for $m = 1$, $z = \perp$) and Axiom 4 (for $m \geq 2$). By convention, extend this to $\bar{Q}(u; m, u) := 0$ for every $u \in X$ and every m.

Placing x first gives $\bar{Q}(x; 1, \perp) = Q_1(x)$, since an alternative placed first has nothing in front of it. The convention $\bar{Q}(u; m, u) := 0$ reflects that an alternative cannot precede itself; it matches the natural limit of $\bar{Q}(x; m, z)$ as the target x approaches z from above, since the interval of thresholds still able to choose x over z then shrinks to empty. For $z \in X$ with $\rho(z) \geq 1$ and $2 \leq m \leq \rho(z) + 1$, write $\tau(m, z) := \bar{Q}(z; m, z^-)$, the probability that z is chosen when placed at position m, preceded only by its own immediate junior.

Axiom 5 (Self-Depth Monotonicity)**.** For every $z \in X$ and every $m \geq 1$ such that $\tau(m, z)$ and $\tau(m+1, z)$ are both defined: $\tau(m, z) \geq \tau(m+1, z)$.

Fix z and imagine it placed just behind its own immediate junior z^- (or, at $m = 1$, with no predecessor at all), at increasingly deep positions in the list. Since z^- is worse than z, z is never dominated by this single predecessor, so being reached at all is only a matter of search depth. Pushing z one position deeper, while its only possible obstacle remains this same harmless predecessor, can only ever make it weakly harder for the decision maker's search to reach that far, so her probability of choosing z there can only weakly fall.

Proposition 16. *Any RDRTM satisfies Axiom 5.*

The proof of Proposition 16 can be found in Appendix B.

Axiom 6 (Cross-Witness Consistency)**.** For every $u \in X$ with $\rho(u) \geq 1$, every $x, w \in X$ with $x \succeq u$ and $w \succeq u$ (using the convention above whenever $x = u$ or $w = u$), and every position $m \in \{1, \ldots, n\}$, whenever the relevant terms are defined:

$$\bar{Q}(x; m, u^-) - \bar{Q}(x; m, u) = \bar{Q}(w; m, u^-) - \bar{Q}(w; m, u).$$

Fix a position m and an alternative u with junior u^-. Whether we watch the effect on some far-better alternative's choice probability, or on u's own (through the convention above), replacing u^- by u as the top predecessor removes exactly the same thing from contention: the possibility that the decision maker's threshold is u itself. This possibility is a feature of u and m alone, not of who is being targeted, so the resulting drop in choice probability must be the same number no matter which undominated alternative above u is used to measure it.

To see this concretely, return once more to the example of Table IV, where $x \succ y \succ z$, and take $u = y$ (so $u^- = z$) and $m = 2$. Lists $L = zxy$ and $L' = yxz$ both place x at position 2, with best predecessor z in L and y in L'; by Proposition 9, $\bar{Q}(x; 2, z) = Q(x, zxy) = \tau(2, x) + \tau(2, y)$, since the eligible interval is $\{y, x\}$, while $\bar{Q}(x; 2, y) = Q(x, yxz) = \tau(2, x)$, since y itself is now excluded from the eligible interval $\{x\}$. Replacing z by y as x's predecessor therefore lowers x's choice probability by exactly $\tau(2, y)$: the one possibility it removes is that the decision maker's threshold is y and her search reaches depth 2. Measuring the same replacement through y instead of x, at list $L'' = zyx$, gives $\bar{Q}(y; 2, z) = Q(y, zyx) = \tau(2, y)$, since the eligible interval is $\{y\}$ alone ($x \succ y$ is excluded), while $\bar{Q}(y; 2, y) = 0$ by convention; so this second drop is $\tau(2, y)$ as well. Both witnesses, x and y itself, recover the same number because that number, $\tau(2, y)$, is a fact about y and the depth 2 alone, not about which better-ranked alternative is used to read it off.

Proposition 17. *Any RDRTM satisfies Axiom 6.*

The proof of Proposition 17 can be found in Appendix B.

Axiom 7 (Top Choice)**.** The $\succ$-best alternative $x^\star$ satisfies $Q_1(x^\star) = 1$.

This axiom says that the best alternative, whenever placed first, is chosen with certainty: no realization of her search leaves the decision maker willing to pass it over or to abstain. It is exactly Proposition 6, restated directly on Q.

Axiom 8 (Zero Dominance)**.** If $x \succ y$ and $L(x) < L(y)$, then $Q(y, L) = 0$.

This axiom says that a worse alternative is never chosen once a better one has already been placed ahead of it: any threshold the worse alternative would clear is also cleared by the better one, so the decision maker's search stops at the better alternative first. It is exactly Proposition 7, restated directly on Q.

Theorem 2. *A stochastic choice function Q over lists is an RDRTM if and only if it satisfies Axiom 3–Axiom 8.*

Necessity is established by Proposition 6, Proposition 7, Proposition 9, Proposition 15, Proposition 16, and Proposition 17. For sufficiency, Axiom 3 recovers $\succ$, and, for each $z \in X$, a short counting argument (Lemma B.1 in Appendix B) shows that z can be some alternative's undominated top predecessor at position m if and only if $m \leq \rho(z) + 2$, and correspondingly that $\tau(m, z)$ can matter for choice at all only when $m \leq \rho(z) + 1$. Within exactly this range, $\tau(m, z)$ is recovered directly, as $\bar{Q}(z; m, z^-)$ for $m \geq 2$ or via the adjacent Q_1-differences at $m = 1$; Axiom 6 then guarantees that this reading agrees with the one obtained by tracking the same drop in choice probability from any better-ranked alternative's perspective, so that $\bar{Q}(x; m, z) = \sum_{t: z \prec t \preceq x} \tau(m, t)$ for every reachable (x, m, z) (Lemma B.2). Axiom 5 makes $\sigma(m, z) := \tau(m, z) - \tau(m + 1, z)$ well defined and nonnegative throughout this range, with the leftover mass $\tau(\rho(z) + 1, z)$ assigned to $\sigma(\rho(z) + 1, z)$ and $\sigma(m, z) := 0$ beyond it, which Lemma B.1 shows is provably immaterial to Q. Axiom 7 pins down $\sum_t \tau(1, t) = 1$, so $\eta(z) = \tau(1, z) > 0$ for every z. Reproducing Q on every list then follows by combining Axiom 4 with Axiom 8 on dominated targets, and with the interval-sum identity above on undominated ones. The full proof can be found in Appendix B.

This proof has a further corollary: it pins down exactly which cells of σ are identified from Q, and confirms that the partial identification of Section 5.3 is not an artifact of the swap procedure used there, but a genuine feature of what choice from lists can reveal.

Corollary 1. *Fix a linear order $\succeq$ on X, and let σ, σ' be two densities on $\{1, \ldots, n\} \times X$, each with $\eta(t), \eta'(t) > 0$ for every $t \in X$, inducing stochastic choice functions Q, Q' via Definition 6. Writing $\tau(m, t) := \sum_{k=m}^{n} \sigma(k, t)$ and $\tau'(m, t) := \sum_{k=m}^{n} \sigma'(k, t)$: $Q = Q'$ if and only if $\tau(m, t) = \tau'(m, t)$ for every $t \in X$ and every $m \leq \rho(t) + 1$.*

The proof of Corollary 1 can be found in Appendix B. In words, choice from lists reveals the tail probabilities $\tau(1,t) \geq \tau(2,t) \geq \cdots \geq \tau(\rho(t)+1,t)$ for each t, but is silent on how the remaining mass $\tau(\rho(t)+1,t)$ is further split across the deeper depths $\rho(t)+1, \rho(t)+2, \ldots, n$: no list ever places t far enough down, relative to how many alternatives it dominates, for that split to matter. The canonical σ constructed in the proof above, which assigns all of this residual mass to depth $\rho(t)+1$ exactly and none beyond it, is therefore only one representative of a whole observational equivalence class of primitives consistent with the same Q.

6 Extension

I now discuss one natural relaxation of the main model, motivated by the same examples that opened the paper. In the RDRTM, every instance of abstention is a *failed search*: the decision maker looks at $k \geq 1$ alternatives, having some threshold t in mind, and none of them clears it. But a shopper can also close the browser tab without scrolling through a single search result, a voter can leave the ballot blank without reading a single name on it, and a dating app user can put the phone down without swiping on anyone that session. These are cases of *non-search*: the decision maker never engages with the list at all, so abstention has nothing to do with the alternatives being offered or their order. The baseline RDRTM, whose search depth k ranges only over $\{1, \ldots, n\}$, cannot distinguish this behavior from a very shallow, unlucky search; the extension below restores the distinction by allowing $k = 0$ as a possible realization of search depth, so that failing to search at all becomes a distinct, and separately identified, source of non-choice. Formally, σ is now a probability distribution over $\{0, 1, \ldots, n\} \times X$, and Definition 6 extends unchanged, with the sum in Equation (2) still ranging over $k \geq 1$, since a decision maker who never searches ($k = 0$) can never choose an alternative in X, regardless of her threshold. Because no realization with $k = 0$ ever enters Equation (2), the threshold drawn during non-search has no behavioral consequence whatsoever: choice data can only ever reveal the total mass $\mu(0) := \sum_{t \in X} \sigma(0,t)$, never how that mass is allocated across $\sigma(0,t)$ for individual $t \in X$. This allocation is therefore not a target of the identification results below, and should be understood as a normalization rather than as a recoverable primitive.

Let $\mu(k) := \sum_{t \in X} \sigma(k,t)$ denote the marginal probability that the decision maker's search depth equals k, for $k = 0, 1, \ldots, n$. In this model, if $x^\star$ is such that $x^\star \succeq x$ for all $x \in X$, and L is any list such that $L(x^\star) = 1$, then:

$$Q(x^\star, L) = \sum_{k=1}^{n} \sum_{t \in \Gamma(\succeq, L, x^\star, k)} \sigma(k,t) = \sum_{k=1}^{n} \mu(k) < 1,$$

and $Q(a^\star, L) = \mu(0)$: even the best alternative, heading the list, is no longer chosen with certainty, since with probability $\mu(0)$ the decision maker never looks at the list to begin with. This is precisely why Axiom 7 (Top Choice) is the one axiom, among the six of Section 5.4, that this richer model does not satisfy as stated: $Q_1(x^\star) = 1 - \mu(0)$, which is strictly less than one whenever non-search occurs with positive probability. None of the model's other content is lost by allowing $k = 0$, however: Axiom 3–Axiom 6 and Axiom 8 continue to hold exactly as before, since their proofs (Appendix B) only ever compare probabilities across lists or across alternatives that are already being searched, and are silent on whether search happens at all; only the requirement that the decision maker searches with certainty is given up.

Nevertheless, this model can be re-written so that it coincides with the RDRTM, allowing me to apply the identification and characterization results of Section 5 directly, rather than redeveloping them from scratch. The idea is simple: since non-search occurs with the same probability $\mu(0)$ regardless of the list faced, it can be separated out and conditioned away, leaving behind an ordinary RDRTM over the residual probability $1-\mu(0)$. The procedure is as follows:

1. First, I identify $x^\star$ as the alternative that maximizes $Q_1(x)$ over $x \in X$.[4]

2. Once $x^\star$ is identified, $\mu(0) = 1 - Q_1(x^\star)$. For every $\{x, L\}$, I then define an RDRTM $\hat{Q}$ as follows:
$$\hat{Q}(a^\star, L) = \frac{Q(a^\star, L) - \mu(0)}{1-\mu(0)}, \quad \hat{Q}(x, L) = \frac{Q(x, L)}{1-\mu(0)}. \tag{7}$$

3. From this RDRTM, I can always identify $\succeq$ and $\hat{\eta}$, and, by Corollary 1, $\hat{\tau}(m,t)$ for every $t \in X$ and every $m \leq \rho(t)+1$; multiplying $\hat{\eta}$ by $1-\mu(0)$ recovers the threshold marginal η conditional on searching. Recovering $\hat{\mu}$ in full, and hence the complete depth marginal μ on $\{1, \ldots, n\}$, requires the Independence condition of Definition 7, exactly as in Section 5.3; without it, only the same partial tails of $\hat{\sigma}$ described by Corollary 1 are identified.

The rescaled function $\hat{Q}$ is itself an RDRTM: dividing out the constant $\mu(0)$ leaves a valid probability distribution over $X \cup \{a^\star\}$ for each list, generated by the same preference $\succeq$ and by the conditional density $\sigma(k,t)/(1-\mu(0))$ over depths $k \geq 1$. Consequently, $\hat{Q}$ satisfies all six axioms of Section 5.4, Axiom 7 included, by Theorem 2; and σ and $\succeq$ are identified from $\hat{Q}$ exactly as in Section 5.3 — fully under Independence, and otherwise only through the partial tails of Corollary 1 — hence from Q and the single scalar $\mu(0)$. In this sense, allowing for non-search is not a different model requiring its own separate treatment, but a one-parameter enrichment of the RDRTM that inherits, rather than complicates, everything Section 5 establishes: the preference is still always identified, the joint distribution of depth and threshold is still identified under Independence, and the only genuinely new object to recover is the single scalar $\mu(0)$, which step 1 above reads directly off the data as one minus the largest head-of-list choice probability, $\mu(0) = 1 - \max_{x \in X} Q_1(x)$.

7 Conclusion

This paper studies choice from lists in an environment where the underlying menu never varies and the only source of observable variation is the order in which the same alternatives are presented, as in a shopper's search results, a voter's ballot, or a dating app's queue. I model the decision maker as searching such a list in order and stopping at the first alternative that clears a threshold in her mind, provided she reaches it before exhausting a limited search capacity. A deterministic version of this procedure, in which the search capacity and threshold are fixed, is a useful benchmark: its primitives are only partially identified from choice, and I exhibit a choice function that satisfies the model's most natural monotonicity property without being generated by any deterministic search-and-threshold procedure at all; nonetheless, I show that two axioms repair this gap and deliver a complete axiomatic characterization of the class of choice functions the deterministic model can generate.

4 This alternative is well defined and unique. Placing any $x \in X$ first in a list L leaves it with no predecessor, so $Q_1(x) = Q(x, L) = \sum_{k=1}^{n} \sum_{t \in X: x \succeq t} \sigma(k,t) = \sum_{t: x \succeq t} \eta(t)$. As x ranges from the $\succeq$-worst to the $\succeq$-best alternative, $\{t : x \succeq t\}$ grows strictly, and $\eta(t) > 0$ for every t, so Q_1 is strictly increasing along $\succeq$ and is therefore uniquely maximized at the $\succeq$-best alternative, with $Q_1(x^\star) = \sum_{t \in X} \eta(t) = \sum_{k=1}^{n} \mu(k) = 1 - \mu(0)$.

Randomizing both primitives resolves this. In the resulting Random Depth Random Threshold Model, the underlying preference is always identified from stochastic choice, and, under an independence condition on the joint distribution of search depth and threshold, so is that entire joint distribution; even without this condition, the constructive proof behind the model's characterization identifies a substantial share of the joint distribution directly, as a by-product, though Corollary 1 shows that a precisely delimited triangular region, asymptotically about half of the joint distribution's degrees of freedom, remains unidentified. I show that this model is fully characterized by six testable axioms on stochastic choice from lists (Axiom 3–Axiom 8), each with a direct behavioral interpretation: the probability of heading a list is well defined and strictly ranks alternatives; an alternative's choice probability depends on its predecessors only through the single best one among them; pushing an undominated alternative deeper behind that same predecessor can only weakly lower its choice probability; the resulting drop in choice probability, when a predecessor improves, is the same number no matter which better-ranked alternative is used to measure it; the best alternative is chosen with certainty whenever it heads the list; and a worse alternative is never chosen once a better one precedes it.

These six axioms are not only necessary but sufficient: a stochastic choice function over lists is generated by some Random Depth Random Threshold Model if and only if it satisfies them (Theorem 2), completing the if-and-only-if characterization of choice from lists under random search and satisficing that the deterministic benchmark could not deliver on its own. Section 6 shows that this characterization is not brittle to one further behaviorally motivated source of abstention: allowing the decision maker to never search the list at all, rather than always searching and failing, costs the model only Axiom 7, and every identification and characterization result of Section 5 carries over once the constant probability of non-search is conditioned away. Future work may consider relaxing the assumption that the set of possible thresholds coincides with the alternative set itself, or that the underlying preference is a linear order rather than a weaker binary relation; either extension would make the model directly applicable to richer list-design settings, such as ranking algorithms that can set a threshold outside the set of items they display.

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A Deterministic Model Proofs

A.1 Proof of Proposition 2

Proof. Suppose that there exists a list L such that $c^{\succeq,k,t}(L) = x$ and $L(y) \leq L(x)$. Since $\succeq$ is a linear order, either $x \succeq y$ or $y \succ x$. Suppose $y \succ x$. Then $c^{\succeq,k,t}(L) = x$ implies that $x \succeq t$ and $L(x) \leq k$. However, $L(y) \leq L(x) \leq k$, and by transitivity of the preference relation, $y \succ t$. This implies that y is an alternative that satisfies the threshold and is displayed before x to the decision maker, so $\min\left(A(\succeq, L, k, t), L\right) = y$, a contradiction. Therefore, it must be the case that $x \succeq y$. $\square$

A.2 Proof of Proposition 3

Proof. Consider two lists L, L' such that $x = c(L)$ and $\{y : L'(y) \leq L'(x)\} \subseteq \{y : L(y) \leq L(x)\}$. First, $x = c(L)$ implies $L(x) \leq k$ and $x \succeq t$. Additionally, by the revelation induced by the model, every $z \in \{y : L(y) \leq L(x)\}$ must satisfy $x \succeq z$. In particular, every $z \in \{y : L'(y) \leq L'(x)\}$, a subset of $\{y : L(y) \leq L(x)\}$, must satisfy $x \succeq z$.

Finally, $\{y : L'(y) \leq L'(x)\} \subseteq \{y : L(y) \leq L(x)\}$ implies $L'(x) \leq L(x)$, otherwise there would be more elements in $\{y : L'(y) \leq L'(x)\}$ than in $\{y : L(y) \leq L(x)\}$. Hence x is such that $L'(x) \leq k$, $x \succeq t$, and every predecessor of x in L' is dominated by it, implying $c(L') = x$. $\square$

A.3 Proof of Proposition 4

Proof. Suppose $c^{\succeq,k,t}(L) = x$ and $L(y) < L(x)$. Since $x = \min(A(\succeq, L, k, t), L)$, $x \succeq t$ and $L(x) \leq k$. As $L(y) < L(x) \leq k$, y also lies within depth k. If $y \succeq t$ held, then $y \in A(\succeq, L, k, t)$ and $L(y) < L(x)$ would imply $\min(A(\succeq, L, k, t), L) = y \neq x$, a contradiction. Hence $t \succ y$, so $y \notin A(\succeq, L', k, t)$ for every list L', since membership in $A(\succeq, L', k, t)$ requires $y \succeq t$, which fails regardless of L'. As $\min(S, L')$ is always an element of S whenever $S \neq \emptyset$, $c^{\succeq,k,t}(L') \neq y$ for every list L'. Hence $y \in N(c)$. $\square$

A.4 Proof of Proposition 5

Proof. Let $c = c^{\succeq,k,t}$ and let $B := \{z \in X : t \succ z\}$. I first show $N(c) = B$. If $z \in B$, then $z \notin A(\succeq, L'', k, t)$ for every list L'', so, exactly as in the proof of Proposition 4, $c^{\succeq,k,t}(L'') \neq z$ for every list L'', i.e., $z \in N(c)$. Conversely, if $z \notin B$, then $z \succeq t$; placing z first in a list L'' gives $z \in A(\succeq, L'', k, t)$ (since $L''(z) = 1 \leq k$) and $z = \min(A(\succeq, L'', k, t), L'')$ trivially, so $c(L'') = z$ and $z \notin N(c)$. Hence $N(c) = B$.

Suppose $c(L) = a^\star$ and $c(L') \neq a^\star$. Since $c(L) = a^\star$, $A(\succeq, L, k, t) = \emptyset$, so every alternative among the first k positions of L fails $z \succeq t$, i.e., $\{L^{-1}(1), \ldots, L^{-1}(k)\} \subseteq B = N(c)$. Hence $\beta(L) \geq k$.

Let $x' := c(L') \neq a^\star$. Then $x' \succeq t$, so $x' \notin N(c)$, and $L'(x') \leq k$. By Proposition 4, every w preceding x' in L' satisfies $w \in N(c)$. Hence $\{L'^{-1}(1), \ldots, L'^{-1}(L'(x') - 1)\} \subseteq N(c)$, while position $L'(x')$ itself holds $x' \notin N(c)$; so the initial run of never-chosen alternatives in L' has length exactly $\beta(L') = L'(x') - 1$. Since $L'(x') \leq k$, $\beta(L') \leq k - 1 < k \leq \beta(L)$. □

A.5 Proof of Theorem 1

Proof. Necessity of Axiom 1 and Axiom 2 is established by Proposition 4 and Proposition 5, respectively.

For sufficiency, suppose c satisfies Axiom 1 and Axiom 2, and $c(L) \neq a^\star$ for at least one list L. Let $G := X \setminus N(c)$; since some list L has $c(L) = x \neq a^\star$, and $x \notin N(c)$ by definition of $N(c)$, $G \neq \varnothing$.

Construction. Fix any linear order $\succeq$ on X that ranks every element of G above every element of $N(c)$ (such an order exists: order G arbitrarily, order $N(c)$ arbitrarily, and rank the first block above the second). Let t be the $\succeq$-worst alternative in G, so that $\{z \in X : z \succeq t\} = G$. Let

$$k := \begin{cases} \min\{\beta(L) : c(L) = a^\star\} & \text{if } c(L) = a^\star \text{ for some list } L, \\ n & \text{otherwise.} \end{cases}$$

$k \in \{1, \ldots, n\}$. If c never abstains, $k = n$. Otherwise, let $\hat{L}$ be an abstaining list with $\beta(\hat{L}) = k$. Since $c(L) \neq a^\star$ for some list L, Axiom 2 gives $\beta(L) < \beta(\hat{L}) = k$, and $\beta(L) \geq 0$, so $k \geq 1$. Since $G \neq \varnothing$, $\beta(\hat{L}) \leq n - |G| \leq n - 1$, so $k \leq n - 1 < n$.

$c^{\succeq,k,t} = c$. Fix a list L. There are two cases.

Case 1: $c(L) = x \neq a^\star$. By Axiom 1, every y with $L(y) < L(x)$ satisfies $y \in N(c)$, and $x \in G$ since $x = c(L) \notin N(c)$. Hence $\{L^{-1}(1), \ldots, L^{-1}(L(x) - 1)\} \subseteq N(c)$, while position $L(x)$ itself holds $x \notin N(c)$, so the initial run of never-chosen alternatives in L has length exactly $\beta(L) = L(x) - 1$. If c abstains at some list, Axiom 2 applied to L and $\hat{L}$ (as defined above, with $\beta(\hat{L}) = k$) gives $\beta(L) < k$; if c never abstains, $\beta(L) \leq n - 1 < n = k$. Either way, $L(x) - 1 = \beta(L) < k$, so $L(x) \leq k$.

Now, $A(\succeq, L, k, t) = \{z \in X : z \succeq t, L(z) \leq k\} = \{z \in G : L(z) \leq k\}$, using $\{z : z \succeq t\} = G$. Since $x \in G$ and $L(x) \leq k$, $x \in A(\succeq, L, k, t)$. Moreover, every z with $L(z) < L(x)$ satisfies $z \in N(c)$, hence $z \notin G$, hence $z \notin A(\succeq, L, k, t)$; so no element of $A(\succeq, L, k, t)$ precedes x in L, and $x = \min(A(\succeq, L, k, t), L) = c^{\succeq,k,t}(L)$.

Case 2: $c(L) = a^\star$. By definition of k, $\beta(L) \geq k$, so $\{L^{-1}(1), \ldots, L^{-1}(k)\} \subseteq N(c)$, i.e., no element among the first k positions of L lies in G. Hence $A(\succeq, L, k, t) = \{z \in G : L(z) \leq k\} = \varnothing$, and $c^{\succeq,k,t}(L) = \min(\varnothing, L) = a^\star = c(L)$.

In both cases $c^{\succeq,k,t}(L) = c(L)$, so $c^{\succeq,k,t} = c$ and c is a DTM. □

B Stochastic Model Proofs

B.1 Proof of Proposition 6

Proof. Since $\succeq$ is a linear order, I can order alternatives from the best to the least preferred according to $\succeq$, $x^\star \succ x_2 \succ x_3 \succ \cdots \succ x_n$. Therefore $x^\star$, with the property that $x^\star \succ x$ for every $x \in X \setminus \{x^\star\}$, always exists.

If L is such that $L(x^\star) = 1$, then whatever the threshold $t \in X$ is, $x^\star \succeq t$. This implies that $x^\star \in A(\succeq, k, t, L)$ for all $k = 1, \ldots, n$, and $L(x^\star) < L(y)$ for all $y \in A(\succeq, k, t, L)$. Therefore:

$$Q(x^\star, L) = \sum_{(k,t):x=c^{\succeq,k,t}(L)} \sigma(k,t) = \sum_{k=1}^{n} \sum_{t \in X} \sigma(k,t) = 1.$$

This last equality holds since σ is a density function over $\{1, \ldots, n\} \times X$. $\square$

B.2 Proof of Proposition 7

Proof. Consider any depth k such that $L(x) < L(y) \leq k$. If t is a threshold such that $y \succeq t$, then $x \in A(\succeq, k, t, L)$ since $x \succ y$. But this implies that $y \neq \min(A(\succeq, k, t, L), L)$, since there is an alternative, x, that is better than t and comes before y. This implies that $y \neq c^{\succeq,k,t}(L)$ for all $k \geq L(y)$. Trivially, if $k < L(y)$ then $y \neq c^{\succeq,k,t}(L)$. Hence $\{(k,t) : y = c^{\succeq,k,t}(L)\} = \emptyset$, which implies:

$$Q(y, L) = \sum_{(k,t):y=c^{\succeq,k,t}(L)} \sigma(k,t) = 0.$$

$\square$

B.3 Proof of Proposition 9

Proof. Fix $x \in X$, a list L, and $k \geq L(x)$. Since $L(x) \leq k$ holds automatically for every such k, the condition "$L(z) < L(x) \leq k$" defining $\Gamma(\succeq, L, x, k)$ reduces to "$L(z) < L(x)$" alone, which does not involve k. Hence $\Gamma(\succeq, L, x, k) = \Gamma(\succeq, L, x)$ for every $k \geq L(x)$, where

$$\Gamma(\succeq, L, x) := \{t : x \succeq t \text{ and } t \succ z \text{ for every } z \text{ with } L(z) < L(x)\}.$$

If $\{y : L(y) < L(x)\} = \emptyset$ this is simply $\{t : x \succeq t\}$, matching the stated convention. Otherwise, let $z^\star = z^\star(x, L)$. Since $z^\star \succeq z$ for every predecessor z of x, "$t \succ z$ for every predecessor z" holds if and only if $t \succ z^\star$: if $t \succ z^\star$ then $t \succ z^\star \succeq z$ for every predecessor z by transitivity, while if $t \succ z$ fails for some predecessor z, it fails in particular for $z = z^\star$, since $z^\star \succeq z$. Hence $\Gamma(\succeq, L, x) = \{t : z^\star \prec t \preceq x\}$.

Using Equation (4) together with this simplification,

$$Q(x, L) = \sum_{k=L(x)}^{n} \sum_{t \in \Gamma(\succeq, L, x, k)} \sigma(k,t) = \sum_{k=L(x)}^{n} \sum_{t \in \Gamma(\succeq, L, x)} \sigma(k,t) = \sum_{t \in \Gamma(\succeq, L, x)} \sum_{k=L(x)}^{n} \sigma(k,t) = \sum_{t \in \Gamma(\succeq, L, x)} \tau(L(x), t),$$

which is Equation (5). $\square$

B.4 Proof of Proposition 10

Proof. Let L, L' be two lists such that, for a given x, $\{y : L'(y) \leq L'(x)\} \subseteq \{y : L(y) \leq L(x)\}$. Then for every k, $\Gamma(\succeq, L, x, k) \subseteq \Gamma(\succeq, L', x, k)$, since the set of predecessors of x in L contains the set of predecessors of x in L', so requiring a threshold t to beat every predecessor of x is a weakly more demanding condition under L than under L'. Additionally, it must be the case that $L'(x) \leq L(x)$, otherwise $\{y : L'(y) \leq L'(x)\}$ would be a larger set than $\{y : L(y) \leq L(x)\}$. This implies:

$$Q(x, L) = \sum_{k=L(x)}^{n} \sum_{t \in \Gamma(\succeq, L, x, k)} \sigma(k, t) \leq \sum_{k=L(x)}^{n} \sum_{t \in \Gamma(\succeq, L', x, k)} \sigma(k, t) \leq \sum_{k=L'(x)}^{n} \sum_{t \in \Gamma(\succeq, L', x, k)} \sigma(k, t) = Q(x, L').$$

□

B.5 Proof of Proposition 11

Proof. Consider $x \neq y$ and two lists L, L' such that $L = x \ldots$ and $L' = y \ldots$. Suppose that $Q(x, L) > Q(y, L')$.

Given that x, y are the first elements in lists L, L' respectively, and therefore have no predecessors, the Γ sets can be rewritten as:[5]

$$\Gamma(\succeq, x, L, k) = \{t : x \succeq t\}, \quad \Gamma(\succeq, y, L, k) = \{t : y \succeq t\},$$

for every $k \in \{1, \ldots, n\}$. This implies that:

$$\begin{aligned} Q(x, L) &= \sum_{k=1}^{n} \sum_{t : x \succeq t} \sigma(k, t) \\ &= \sum_{t : x \succeq t} \sum_{k=1}^{n} \sigma(k, t) \\ &= \sum_{t : x \succeq t} \eta(t). \end{aligned}$$

Similarly, $Q(y, L') = \sum_{t : y \succeq t} \eta(t)$. Now, given that $\succeq$ is a linear order, either $x \succ y$ or $y \succ x$. Suppose $y \succ x$. This would imply that $\{t : x \succeq t\} \subset \{t : y \succeq t\}$, since, by transitivity of $\succeq$, every element t with $x \succeq t$ is also less preferred than y. Therefore:

$$Q(x, L) = \sum_{t : x \succeq t} \eta(t) < \sum_{t : y \succeq t} \eta(t) = Q(y, L').$$

However, this contradicts $Q(x, L) > Q(y.L')$. Therefore, it must be the case that $x \succ y$. □

5 Recall that, given a depth search parameter k, $\Gamma(\succeq, L, x, k) = \{t : x \succeq t \text{ and } t \succ z \text{ if } L(z) < L(x) \leq k\}$.

B.6 Proof of Proposition 12

Proof. Fix $i \in \{1, \ldots, n\}$ and a list L with $L(x_i) = 1$. Since x_i has no predecessor in L, Proposition 9 gives

$$Q_1(x_i) = Q(x_i, L) = \sum_{t\,:\,t \preceq x_i} \tau(1,t) = \sum_{t\,:\,t \preceq x_i} \eta(t) = \sum_{j=i}^{n} \eta(x_j),$$

using $\tau(1,t) = \sum_{k=1}^{n} \sigma(k,t) = \eta(t)$ and $\{t : t \preceq x_i\} = \{x_i, x_{i+1}, \ldots, x_n\}$, since $x_1 \succ x_2 \succ \cdots \succ x_n$.

For $i < n$, $Q_1(x_i) - Q_1(x_{i+1}) = \sum_{j=i}^{n} \eta(x_j) - \sum_{j=i+1}^{n} \eta(x_j) = \eta(x_i)$. For $i = n$, $Q_1(x_n) = \eta(x_n)$ directly. $\square$

B.7 Proof of Proposition 13

Proof. Let $\{x, y\}$ be such that $x \succ y$ and there exists no w with $x \succ w \succ y$. Consider two lists L, L' such that $L = \ldots xy \ldots$, $L' = \ldots yx \ldots$ and, in both, every predecessor w of x satisfies $x \succeq w$.

First, for every $z \in X \setminus \{x, y\}$, $\{w : L(w) < L(z)\} = \{w : L'(w) < L'(z)\}$, since swapping x and y does not change the set of predecessors of an alternative z with $L(z) > L(y)$ (similarly if $L(z) < L(y)$). This implies $Q(z, L) = Q(z, L')$ for all $z \in X \setminus \{x, y\}$. In this sense, the probability of abstention potentially differs between lists L and L' only due to the choice probability of $\{x, y\}$ in each list.

Now, in list L (where $L(x) = m$), $x \in \Gamma(\succeq, L, x, m)$, since $x \succeq t = x$ and $t = x \succ w$ for all w with $L(w) < L(x) \leq m$ (by assumption, together with $w \neq x$). However, $x \notin \Gamma(\succeq, L', x, m)$, since $L'(x) = m + 1$. This implies that there is a possibility for abstention in list L' not present in list L: if the decision maker observes the first m elements of L' (where $L'(y) = m$) and has threshold $t = x$ in mind, then $A(\succeq, m, x, L') = \emptyset$, so $\min(A(\succeq, m, x, L'), L') = a^\star$. Therefore, the probability of abstention in L', relative to L, increases by $\sigma(m, x)$.

Finally, $\Gamma(\succeq, L, x, k) = \Gamma(\succeq, L', x, k)$ for all $k \geq m + 2$ (similarly for y), which implies that if the decision maker observes more than $m + 1$ elements, the abstention probability conditional on this remains the same in both lists. Hence the only difference between $Q(a^\star, L')$ and $Q(a^\star, L)$ is $\sigma(m, x)$, which implies:

$$\sigma(m, x) = Q(a^\star, L') - Q(a^\star, L).$$

$\square$

B.8 Proof of Proposition 14

Proof. Since $\succeq$ is a linear order, there exists z such that $x \succ z$ for all $x \in X \setminus \{z\}$. Consider any list L, and let $m = L(z)$. Consider y, the alternative with $L(y) = m + 1$. If L' is the list that only swaps z, y, then in list L', $Q(z, L') = 0$, since there exists an element (y) with $y \succ z$ and $L(y) < L(z)$. In this sense, the abstention probability does not change between L and L', making it impossible to identify $\sigma(m, z)$. $\square$

B.9 Proof of Proposition 15

Proof. By Proposition 9, for any list L with $L(x) = 1$, $Q(x, L) = \sum_{t \preceq x} \tau(1, t)$, which does not depend on L beyond the position of x; write $Q_1(x)$ for this value. Since $x \in \{t : t \preceq x\}$, $Q_1(x) \geq \tau(1, x) = \eta(x) > 0$. For $x \neq y$, since $\succeq$ is a linear order, suppose without loss of generality $x \succ y$; then $\{t : t \preceq y\} \subsetneq \{t : t \preceq x\}$ (both are downsets, and x belongs to the latter but, by antisymmetry, not to the former), so

$$Q_1(x) - Q_1(y) = \sum_{t\,:\,y \prec t \preceq x} \tau(1, t) \geq \tau(1, x) = \eta(x) > 0,$$

using $x \in \{t : y \prec t \preceq x\}$. Hence $Q_1(x) \neq Q_1(y)$. $\square$

B.10 Proof of Proposition 16

Proof. Fix $z \in X$ and $m \geq 1$ such that $\tau(m, z), \tau(m+1, z)$ are both defined. If $m \geq 2$: by Proposition 9, since the eligible interval $\{t : z^- \prec t \preceq z\}$ equals $\{z\}$, $\bar{Q}(z; m, z^-) = \sum_{k \geq m} \sigma(k, z)$, and likewise at $m + 1$; since $\sigma \geq 0$, this tail sum is non-increasing in m. If $m = 1$: by Proposition 9 applied to z and to z^-, each placed first, $Q_1(z) = \sum_{t \preceq z} \tau(1, t)$ and $Q_1(z^-) = \sum_{t \preceq z^-} \tau(1, t)$; since $\{t \preceq z\} \setminus \{t \preceq z^-\} = \{z\}$,

$$\tau(1, z) := Q_1(z) - Q_1(z^-) = \sum_{k=1}^{n} \sigma(k, z),$$

the model's own mass at depth 1, so the same tail-sum argument gives $\tau(1, z) \geq \tau(2, z)$. $\square$

B.11 Proof of Proposition 17

Proof. Fix $u \in X$ with junior u^-, and a position m. For any $x \succeq u$ with $\bar{Q}(x; m, u^-)$ and $\bar{Q}(x; m, u)$ both defined: if $x \succ u$, Proposition 9 gives $\bar{Q}(x; m, u^-) = \sum_{t: u^- \prec t \preceq x} \tau(m, t)$ and $\bar{Q}(x; m, u) = \sum_{t: u \prec t \preceq x} \tau(m, t)$; since $\{t : u^- \prec t \preceq x\} = \{t : u \prec t \preceq x\} \cup \{u\}$, the difference of the two is $\tau(m, u)$, independent of x. If $x = u$, the convention $\bar{Q}(u; m, u) := 0$, together with $\bar{Q}(u; m, u^-) = \tau(m, u)$ (the singleton-interval case of Proposition 9), gives the same difference $\tau(m, u)$. Hence the common value is $\tau(m, u)$ regardless of the witness $x \succeq u$ used to compute it. $\square$

B.12 Two Lemmas for Theorem 2

Lemma B.1 (Reachability). *Let $z \in X$ have $\succ$-rank j, so $\rho(z) = n - j$.*

(a) For $x \succ z$ and $m \geq 2$, a list L with $L(x) = m$ and $\succ$-best predecessor equal to z exists if and only if $2 \leq m \leq \rho(z) + 2$.

(b) Consequently, $\bar{Q}(z; m, z^-)$ is defined if and only if $2 \leq m \leq \rho(z) + 1$; and, for any reachable configuration (x, m, z'), the eligible interval of $\bar{Q}(x; m, z')$ can contain z only if $m \leq \rho(z) + 1$.

Proof. (a) ($\Rightarrow$) If such an L exists, x's $m - 1$ predecessors consist of z together with $m - 2$ further alternatives, each strictly worse than z (otherwise z would not be the best among them); since exactly $\rho(z)$ alternatives are strictly worse than z, $m - 2 \leq \rho(z)$. ($\Leftarrow$) If $2 \leq m \leq \rho(z) + 2$, choose any $m - 2$ alternatives strictly worse than z (possible since $m - 2 \leq \rho(z)$), place them together with z, in any order, in the first $m - 1$ positions of L, place x in position m, and place every remaining alternative afterward; then z is the $\succ$-best of x's $m-1$ predecessors.

(b) The first claim is (a) applied with $x = z$ and predecessor z^-, whose rank is $j+1$, so $\rho(z^-) = \rho(z) - 1$ and the bound $m \le \rho(z^-) + 2$ reads $m \le \rho(z) + 1$. For the second claim, z lies in the eligible interval $\{t : z' \prec t \preceq x\}$ of $\bar{Q}(x; m, z')$ only if $z' \prec z$, i.e. z' has rank at least $j+1$, so $\rho(z') \le \rho(z) - 1$; by (a) applied to z', reachability of (x, m, z') requires $m \le \rho(z') + 2 \le \rho(z) + 1$. $\square$

Lemma B.2 (Telescoping). *Suppose Q satisfies Axiom 3–Axiom 6. For every $x \in X$ and $z \prec x$ with (x, m, z) reachable (Lemma B.1):*

$$\bar{Q}(x; m, z) = \sum_{t \,:\, z \prec t \preceq x} \tau(m, t).$$

The same holds at $z = \perp$, $m = 1$: $\bar{Q}(x; 1, \perp) = Q_1(x) = \sum_{t \preceq x} \tau(1, t)$.

Proof. The case $z = \perp$ is immediate by telescoping the defining differences of $\tau(1, \cdot)$: writing $x = x_i$ in the order $x_1 \succ \cdots \succ x_n$,

$$\sum_{t \preceq x_i} \tau(1, t) = \sum_{l=i}^{n} \tau(1, x_l) = [Q_1(x_i) - Q_1(x_{i+1})] + \cdots + [Q_1(x_{n-1}) - Q_1(x_n)] + Q_1(x_n) = Q_1(x_i).$$

For $z \ne \perp$: list the alternatives strictly worse than x, in decreasing order, as $y_1 = x^-, y_2 = y_1^-, \ldots, y_r$ (where $r = \rho(x)$). I show by induction on i that $\bar{Q}(x; m, y_i) = \tau(m, x) + \sum_{l=1}^{i-1} \tau(m, y_l)$ for every $i = 1, \ldots, r$ and every $m \le \rho(y_i) + 2$.

Base case $i = 1$. By definition, $\tau(m, x) := \bar{Q}(x; m, x^-) = \bar{Q}(x; m, y_1)$, defined precisely for $2 \le m \le \rho(x) + 1 = \rho(y_1) + 2$, matching the claimed range.

Inductive step. Suppose the claim holds for i, and let $m \le \rho(y_{i+1}) + 2 = \rho(y_i) + 1$ (using $\rho(y_i) = \rho(y_{i+1}) + 1$). Then: $\tau(m, y_i) := \bar{Q}(y_i; m, y_i^-) = \bar{Q}(y_i; m, y_{i+1})$ is defined (Lemma B.1(b), since $m \le \rho(y_i) + 1$); (x, m, y_i) is reachable, since $m \le \rho(y_i) + 1 < \rho(y_i) + 2$; and (x, m, y_{i+1}) is reachable, since $m \le \rho(y_{i+1}) + 2$. Applying Axiom 6 with $u = y_i$ (so $u^- = y_{i+1}$) and witnesses x and y_i (the latter via the convention $\bar{Q}(y_i; m, y_i) := 0$):

$$\bar{Q}(x; m, y_{i+1}) - \bar{Q}(x; m, y_i) = \bar{Q}(y_i; m, y_{i+1}) - \bar{Q}(y_i; m, y_i) = \tau(m, y_i) - 0 = \tau(m, y_i).$$

Adding the inductive hypothesis gives $\bar{Q}(x; m, y_{i+1}) = \tau(m, x) + \sum_{l=1}^{i} \tau(m, y_l)$, which, since $\{t : y_{i+1} \prec t \preceq x\} = \{x, y_1, \ldots, y_i\}$, is exactly the claim for $i+1$. $\square$

B.13 Proof of Theorem 2

Proof. **Necessity.** Axiom 3 is established in Proposition 15, above; Axiom 4 is Proposition 9; Axiom 5 is established in Proposition 16, above; Axiom 6 is established in Proposition 17, above; Axiom 7 is Proposition 6; and Axiom 8 is Proposition 7.

Sufficiency. Suppose Q satisfies Axiom 3–Axiom 8. By Axiom 3, Q_1 is well-defined, strictly positive, and injective; define $x \succ y \iff Q_1(x) > Q_1(y)$, automatically a linear order on the finite set X since it is induced by an injective real-valued map, and order X as $x_1 \succ x_2 \succ \cdots \succ x_n$, so $\rho(x_i) = n - i$. By Axiom 4, $\bar{Q}(x; m, z)$ is well-defined for every reachable configuration.

Constructing τ. For $i < n$, let $\tau(1, x_i) := Q_1(x_i) - Q_1(x_{i+1})$, and $\tau(1, x_n) := Q_1(x_n)$; by telescoping, $\sum_{i=1}^{n} \tau(1, x_i) = Q_1(x_1) = 1$ by Axiom 7, and each $\tau(1, x_i) > 0$ by the same argument as in Proposition 15. For $i < n$ and $2 \leq m \leq n - i + 1$, let $\tau(m, x_i) := \bar{Q}(x_i; m, x_i^-)$, well-defined by Lemma B.1(b).

Constructing σ. For $i < n$: for $1 \leq m \leq n - i$, set $\sigma(m, x_i) := \tau(m, x_i) - \tau(m + 1, x_i) \geq 0$, by Axiom 5; set $\sigma(n - i + 1, x_i) := \tau(n - i + 1, x_i)$; and set $\sigma(m, x_i) := 0$ for $m > n - i + 1$. For x_n, set $\sigma(1, x_n) := \tau(1, x_n)$ and $\sigma(m, x_n) := 0$ for $m \geq 2$. In every case, telescoping gives $\sum_{m=1}^{n} \sigma(m, x_i) = \tau(1, x_i)$, so σ is a probability density on $\{1, \ldots, n\} \times X$ with $\eta(t) = \tau(1, t) > 0$ for every $t \in X$: $(\succ, \sigma)$ satisfies Definition 6.

Reproduction. Fix a list L and $x \in X$, and let $z^\star = z^\star(x, L)$ be x's best predecessor in L under $\succ$ (or $\perp$, if none). If $z^\star \neq \perp$ and $z^\star \succ x$, Axiom 8 gives $Q(x, L) = 0$, matching the RDRTM $(\succ, \sigma)$ just constructed, whose eligible interval $\{t : z^\star \prec t \preceq x\}$ is empty in this case (as $z^\star \succ x$ rules out any t with both $z^\star \prec t$ and $t \preceq x$). Otherwise ($z^\star = \perp$ or $z^\star \prec x$), Axiom 4 gives $Q(x, L) = \bar{Q}(x; L(x), z^\star)$, which by Lemma B.2 equals $\sum_{t: z^\star \prec t \preceq x} \tau(L(x), t)$, exactly Equation (5) for the constructed RDRTM. Since L and x were arbitrary, $(\succ, \sigma)$ reproduces Q on every list, so Q is an RDRTM. $\square$

B.14 Proof of Corollary 1

Proof. Order X as $x_1 \succ \cdots \succ x_n$, so $\rho(x_i) = n - i$.

($\Rightarrow$) Suppose $Q = Q'$, so in particular $Q_1 = Q_1'$. Fix $i \in \{1, \ldots, n\}$. For $m = 1$: Proposition 12, applied to σ and then to σ', gives $\tau(1, x_i) = \eta(x_i)$ and $\tau'(1, x_i) = \eta'(x_i)$ by the same formula in $Q_1 = Q_1'$ alone (the difference $Q_1(x_i) - Q_1(x_{i+1})$, or $Q_1(x_n)$ if $i = n$); hence $\tau(1, x_i) = \tau'(1, x_i)$.

For $2 \leq m \leq \rho(x_i) + 1$ (so $i < n$): applying Lemma B.1(a) with $z = x_i^-$ and target x_i (note $x_i \succ x_i^-$, and $\rho(x_i^-) = \rho(x_i) - 1$, so the bound $2 \leq m \leq \rho(x_i^-) + 2$ reads $2 \leq m \leq \rho(x_i) + 1$), there exists a list L with $L(x_i) = m$ and best predecessor x_i^-. Since no alternative lies strictly between x_i^- and x_i, Proposition 9 applied to σ gives $Q(x_i, L) = \sum_{t: x_i^- \prec t \preceq x_i} \tau(m, t) = \tau(m, x_i)$; the same list L is reachable regardless of the density used, since reachability depends only on $\succeq$ (Lemma B.1(a)), so Proposition 9 applied to σ' gives $Q'(x_i, L) = \tau'(m, x_i)$. Since $Q(x_i, L) = Q'(x_i, L)$, $\tau(m, x_i) = \tau'(m, x_i)$.

($\Leftarrow$) Suppose $\tau(m, t) = \tau'(m, t)$ for every $t \in X$ and every $m \leq \rho(t) + 1$. Fix a list L and $x \in X$, and let $z^\star = z^\star(x, L)$ (or $\perp$ if x has no predecessor in L). If $z^\star \neq \perp$ and $z^\star \succ x$, then $Q(x, L) = 0 = Q'(x, L)$ by Proposition 7, applied to σ and to σ'. Otherwise, Proposition 9 gives $Q(x, L) = \sum_{t: z^\star \prec t \preceq x} \tau(L(x), t)$ and $Q'(x, L) = \sum_{t: z^\star \prec t \preceq x} \tau'(L(x), t)$. The configuration $(x, L(x), z^\star)$ is realized by L itself, so by Lemma B.1(b), every t in $\{t : z^\star \prec t \preceq x\}$ satisfies $L(x) \leq \rho(t) + 1$; by hypothesis $\tau(L(x), t) = \tau'(L(x), t)$ for every such t, so $Q(x, L) = Q'(x, L)$. Since x and L were arbitrary, $Q = Q'$. $\square$