Mexico's Recent Inflation History as a Result of Fiscal Deficits and its Expectations*

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Abstract

I present a hidden Markov framework that allows me to estimate the historical relationship between fiscal policy, inflationary expectations, and observed inflation for Mexico's case during 1969 and 2016. Due to a data availability problem with the historical sequences about fiscal policy and inflation expectations, I replicate the model presented in Sargent et al. (JPE, 2009), a paper that proposes a parsimonious theoretical framework that explains Mexico's recent inflation history as a function of an estimated sequence of these variables. Therefore, the main results of this paper can be seen as an accounting exercise in which I am decomposing inflation into two components (fiscal and expected driven inflation) that are not orthogonal. The estimation suggests that deficit levels above 5% have lead to the several high inflation episodes this country has experienced. Additionally, the role of expectations has been crucial since a 1% increase in them has lead to a 0.56 percentage points inflation rise, on average. As a way to validate the model's predictions, I contrast the sequences generated by the model with the available data and observe an acceptable fit. Finally, as a contribution of this work, considering the estimated historical relationship, I present several forecasts of the 2017-2021 inflation rate that are function of potential fiscal deficit paths. These forecasts suggest that, in order to maintain a controlled inflation during the upcoming years, the Fiscal Authority should produce a moderate fiscal deficit.

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1 Introduction

The international experience of the second half of the twentieth century suggests that high and hyperinflation episodes commonly occur when an economy presents considerable fiscal deficits.[1] [2] Whenever these episodes presented themselves, usually a fiscal dominance regime prevailed: the Monetary Authority adjusted its policy in order to make the fiscal path sustainable. Under fiscal dominance, the government's deficit has an effect on inflation since these imbalances between public revenue and expenditures are financed with money creation; and, if a Central Bank continues to fund an increasing deficit, a high or even an hyperinflation episode will likely occur. Furthermore, as highlighted in the seminal paper by Cagan (1956), expectations play a central role in high inflation episodes since they reinforce inflation's inertia and cause an even larger inflation spike. Mexico has not been the exception: between 1960 and 2016, there have been several years in which the annual inflation rate exceeded 50% and, as documented by several authors, these episodes were partially caused by considerably high fiscal deficits together with an increase of expectations.[3]

In this paper I develop a model to study the relationship between inflation, its expectations, and fiscal deficits in Mexico between 1969 and 2016.[4] The research questions I aim to answer are: Which fiscal deficit levels have induced certain dynamics between observed and expected inflation that led to high inflation episodes in the case of Mexico? What does this relationship imply for the upcoming years? Studying the relationship between fiscal policy and inflation has several complications due to data availability. In particular, fiscal related variables are not always available or have frequent methodology changes.[5] Therefore, establishing an empiric relationship between inflation (or other macroeconomic variables) and fiscal policy is not always trivial. Hence, to answer these questions, I consider the framework presented in Sargent et al. (JPE, 2009), a paper that studies fiscal dominance regimes that caused high/hyperinflation in five Latin American countries (Argentina, Bolivia, Brazil, Chile, and Peru) and that proposes a novel methodology that deals with the data availability problem. This framework allows me to explain the evolution of inflation as a function of an estimated sequence of inflationary expectations and fiscal deficits. Therefore, I assess which fiscal deficit levels induced high inflation in Mexico according to this methodology.

Sargent et al. (2009) propose a non-linear hidden Markov framework with the following main components: (i) a real balances demand as in Cagan (1956); (ii) agents that use an adaptive expectations algorithm to update their beliefs on future inflation;[6] (iii) a government budget constraint that relates fiscal deficit with monetary emission; and (iv) a stochastic fiscal deficit whose dynamics are governed by a hidden Markov process. These equations imply an inflation rate that is an increasing function of fiscal deficit and inflationary expectations. The fiscal deficit sequence is considered ex-

1 Fischer et al. (2001) defines high/hyperinflation whenever the annual/monthly inflation rate is above 50%.

2 Several empirical studies like Fischer et al. (2001), Catao and Terrones (2005), and Lin and Chu (2013) document, for more than 100 countries, a relationship between inflation, fiscal deficits, and money creation.

3 For example, Cárdenas (2015) and Meza (2017). In Section 3 I elaborate on Mexico's economic history during 1960 and 2016.

4 I consider fiscal deficit as the primary deficit (the difference between public expenditures and income) net of debt emissions (considering all the interests that must be paid for the government's debt).

5 Before 1990, the fiscal deficit series provided by Mexico's Minister of Finance is an historical construction considering the limited data on primary deficit and financial cost of debt during these years. Additionally, since the perception of what constitutes fiscal responsibility for the government has evolved in recent years, there have been several methodology changes that made the fiscal deficit measurement wider in order to follow international standards (e.g., those of the International Monetary Fund). In Section 5 I elaborate on the difficulties that the data on fiscal related variables present.

6 Agents have adaptive or backward-looking expectations when these are formed by extrapolating past values of the variable being predicted. In Appendix A I discuss, in the context of Sargent et al. (2009)'s model, the different implications of supposing an adaptive or a rational expectations algorithm for agents' beliefs on future inflation.

ogenous, consequently, the model assumes a fiscal dominance regime.[7] The stochastic process that determines fiscal deficits is characterized by a median and variance parameters that can vary over time. Hence, the model allows fiscal deficits to be controlled, moderate, or considerably high. This characteristic induces a non-linear relationship between inflation, expectations, and fiscal deficits, a feature that has been widely documented in the empirical literature about the effect of fiscal policy on inflation.[8]

I consider the relationship that the model induces between inflation, expectations, and fiscal deficits to construct a probability density function for a T period inflation sequence. Then, using Mexican inflation data and taking the inflation density as a likelihood function, I use a block-wise maximum likelihood algorithm to estimate all the parameters involved in the model. Using the estimation results, I make inference on the density function of fiscal deficits and expectations conditional on Mexico's historical inflation in order to construct the sequence of deficits and expectations that are consistent with the inflation data. I interpret these results as the historical relationship between the variables involved in the model. In this sense, the main results of this paper can be seen as an accounting exercise in which I am decomposing inflation into two components (fiscal and expected driven inflation) that are not orthogonal.

The estimation suggests that, in the case of Mexico, when fiscal deficit relative to output has been controlled it represented approximately 2.78% of GDP and it has induced an inflation rate around 3.54%. Whenever fiscal deficit has been at a moderate level (4.76% of GDP) inflation was on average 17.53%. Meanwhile, a high fiscal deficit (9.12% of GDP) has been associated with the highest inflation episodes that Mexico had during the past 48 years, with an average of 79.41%. Regarding inflationary expectations, the model suggests they are crucial to determine inflation: a 1 percentage point (p.p.) increase of the expected inflation rate translates into a 0.56 p.p. inflation rise. Hence, according to the model, a necessary condition to have a stable inflation rate is to anchor inflation expectations. In order to achieve a relatively stable expected inflation rate, the model's estimation implies that inflation should be controlled for several consecutive periods, which can be achieved if fiscal deficit is reduced.

Regarding the high inflation episodes that Mexico had during 1969-2016, the model makes the following predictions: (i) The first high inflation episode started in 1982, year in which fiscal deficit relative to output went from 4.9% to 9.4%. As a consequence, inflation expectations de-anchored and inflation was approximately 80% at the end of 1983. (ii) To control this high inflation, the model suggests that the government made a temporal reduction of its deficit, although, this temporary cut was not enough to anchor inflation expectations and inflation. Therefore, by 1985, inflation was once again 80% and above 100% by 1987. (iii) According to the model, during 1988-1993, an important fiscal reform took place. By 1993, fiscal deficit relative to output was 2.9% of GDP and inflation, together with its expectations, were below 20%. (iv) During 1995, the year of another important crisis, the model predicts a considerable fiscal deficit rise and, consequently, of inflation. (v) Finally, between 2000-2016 the model suggests that Mexico is now in a permanent low deficit regime that has allowed expectations and inflation to be controlled and near *Banco de México*'s inflation target.

To validate the model's estimated sequences for fiscal deficit and expectations, among their relationship with inflation, I contrast them with the available data and observe an acceptable fit: the mean relative error between the model's prediction about expectations/ fiscal deficit and the data is 14.1 / 39.7 %, respectively. The most notable differences between the data and the model's fiscal

7 In Section 6.4 I discuss the interpretation that the model gives to *Banco de México*'s autonomy, which was granted in 1994.

8 For example, consult Catao and Terrones (2005) and Lin and Chu (2013).

deficit sequence occur between 1994-1997, years in whose an inflation crisis occurred due to non-fiscal related variables.[9] Nevertheless, the model attributes this inflation spike to a considerable increase in the fiscal deficit level.

Finally, I present forecasts for the inflation rate between 2017-2021 considering that the median and variance parameters that govern fiscal deficit could change during these years. According to the forecasts, if the Fiscal Authority produces an increasing deficit path during the upcoming years, inflation and expectations could destabilize leading to an inflation rate above *Banco de México*'s target. As suggested by the vast literature about the interactions between fiscal and monetary policy, if agents observe a high fiscal deficit that generates an increasing debt, they could anticipate an aid from the Central Bank to the Fiscal Authority and change their current inflationary expectations, causing a potential inflation spike.[10] Therefore, even though Mexico is not in a fiscal dominance regime any more (as assumed in the model) the forecasts I present may reflect the potential inflation rates should the fiscal deficits enlarge between 2017-2021. Consequently, this model suggests that in order to maintain a controlled inflation within the Central Bank's target, the Fiscal Authority should promote an effective management of the public finances that induces a moderate fiscal deficit.

This paper proceeds as follows: Section 2 reviews part of the literature of high inflation models among some papers about the different inflation crises Mexico has had. Section 3 presents a brief history of Mexico's inflation rate and fiscal deficits during 1960-2016 to provide a context for the model and its predictions. In Section 4, I present the framework described in Sargent et al. (2009) and its key assumptions. The procedure that I follow to estimate the parameters is explained in Section 5. Section 6 presents the main results of the paper: the relationship induced by the model between fiscal deficits, expected, and observed inflation for Mexico during 1969-2016. Additionally, I contrast these predictions with the available data on these variables and make a brief discussion of *Banco de México*'s autonomy among its implications. Section 7 presents several predictions for the inflation rate of 2017-2021 considering alternative paths for the fiscal deficit during these years. Section 8 concludes.

2 Related Literature

High inflation episodes and their causes have been an important part of the monetary literature during the second half of the twentieth century. Cagan (1956) was one of the first papers that presented a simple framework to study hyperinflation dynamics. This paper was able to explain several European hyperinflation episodes by assuming a functional form for the real balances demand and adaptive expectations. According to this model, since agents expect a lower return of money in the future during high inflation episodes, their real balances demand falls, inducing a higher inflation rate. Furthermore, expectations about the future price level increase as inflation grows, which itself generates a higher inflation rate. In this framework expectations play a central role during high inflation episodes since they reinforce inflation's ascending inertia. The model I present in this paper derives from Cagan's framework, although it extends it by considering the influence that fiscal deficits have on inflation and expectations. In this sense, controlled deficits lead to low inflation rates, and high deficits could cause an observed and expected inflation spiral, as in Cagan (1956).

9 Calvo and Mendoza (1996), Cole and Kehoe (1996), and Sachs et al. (1995) analyze the causes and consequences of this crisis.

10 Some papers that highlight the importance of fiscal and monetary policy coordination in order to induce macro stability are Cochrane (2001), Kocherlakota (2012), Sims (2016), and Bianchi and Ilut (2017).

Cagan's seminal paper has been baseline of several fiscal dominance hyperinflation models in whose the fiscal policy decisions influence the monetary base and, consequently, inflation. A central question in these type of models is the role of inflation expectations and the process that agents follow to determine them. For example, Sargent and Wallace (1973) used the fundamental equations of Cagan's model and studied the effects of assuming agents are rational when forming their expectations.[11] Other rational expectations extensions to Cagan's model are Christiano (1987), and Barbosa et al. (2006). Marcet and Nicolini (2003) present a bounded rationality model that incorporates the exchange rate with a real balances demand function and a government budget constraint as in Cagan (1956).[12] The authors prove under which scenarios bounded rationality approximates the inflation dynamics induced by rational expectations. In general, it can be shown that the algorithm used to update the public's expectations on inflation (rationally or adaptive) generate long-run (or steady-state) inflation rates that are similar. However, this assumption has considerable consequences in the out of steady-state dynamics. To provide further insight, Appendix A discusses the differences in the out of steady-state dynamics induced by adaptive and rational expectations in the context of the model considered in this paper.

Another important part of the high inflation literature studies the possible channels through which the Fiscal Authority could influence the price level even with an independent Central Bank (no fiscal dominance), regime in which Mexico has been since 1994. For example, according to Sims (2016), if the public debt is high then the Central Bank will have an impediment to use a crucial instrument to reduce inflation since, by manipulating the interest rate, it could substantially increase the government's debt and induce a default. High inflation reduces the nominal value of debt so, as suggested by Leeper (2013), when the public debt is elevated, the Fiscal Authority could pressure the Central Bank for it to allow a higher inflation, jeopardizing its autonomy. Cochrane (2001) and Kocherlakota (2012) argue that, given an increasing public debt that could be defaulted, agents might foresight an aid from Central Bank to the Fiscal Authority in order to produce a sustainable fiscal policy. Hence, they could increase their current inflation expectations, causing a potential inflation spiral.

Additionally, there are several models that incorporate the possibility of a regime change between Central Bank independence and fiscal dominance. In this class of models, fiscal dominance can be considered as an endogenous result of a Central Bank autonomy regime mainly caused by an inadequate management of public finances. For example, in the classic paper by Sargent and Wallace (1981), the authors develop a model that allows for a regime change from monetary to fiscal dominance. In this model, if the government has an increasing primary deficit (defined as the difference between the government's revenue and expenditures) that overpasses the seigniorage level then the debt to GDP ratio will begin to grow and, eventually, the public will not finance this increasing debt. In this situation, the only way to make the fiscal path sustainable is to have a regime change to fiscal dominance where the government's debt can be financed through money creation. Bianchi and Ilut (2017) and Cadavid-Sanchez et al. (2017) are other examples of these type of models, although, they also consider the influence that these regime changes could have on inflationary expectations.

In general, the consensus in the literature about high inflation episodes is that, with or without fiscal dominance, the fiscal and monetary policy should be coordinated to achieve a low and stable inflation rate. Therefore, this vast literature warns that, even though Mexico has an autonomous Central Bank, the Fiscal Authority should conduct a prudent policy that induces sustainable deficit path. In this sense, some of the predictions that I present for the 2017-2021 inflation rate may be

11 According to Sargent and Wallace (1973): "Expectations about a variable are rational if they depend on the same things that economic theory says actually determine that variable".

12 These authors define bounded rationality as a learning mechanism that has a "small" deviation from rationality.

considered as a warning of what could occur to inflation should the government produce a deficit path that is unsustainable in the future.

Finally, this paper is complementary to several studies about different inflation crises Mexico had between 1950 and 2016. For example, Rogers and Wang (1994) use a structural VAR to assess the effect of several macro-related shocks on inflation between 1977 and 1990. The main conclusion of this paper is that, during the 80s, the fiscal and monetary shocks can account for 60% of the inflation's variance. This result suggests that the inflation crises presented during these years were primarily driven by shocks on these variables. Meza (2017) analyses the inflationary crises presented in 1982 and 1994 using as framework the model of Sargent and Wallace (1981). The author concludes that this model explains the inflationary crisis of 1982, however, it fails to explain the 1994 high inflation episode because it was primarily driven by non-fiscal factors. Calvo and Mendoza (1996), Sachs et al. (1995), and Cole and Kehoe (1996) are other papers that study the crisis presented in Mexico during 1994-1995. There are other interesting papers about Mexican crises like Kehoe and Meza (2012), in which the authors explain several crises that occurred between 1950 and 2010 parting from a neoclassical growth model's perspective.

3 Mexico's Recent Inflation and Fiscal Deficits

As in other Latin American countries, during the second half of the twentieth century, Mexico had several episodes in whose the annual inflation rate was considerably elevated. Figure I presents the annual inflation rate computed with the *Índice Nacional de Precios al Consumidor* (INPC) between 1969 and 2016.[13] According to Fischer et al. (2001)'s definition, this country had high inflation between 1982-1988 and in 1995.

As documented by several authors, these high inflation episodes were usually accompanied by an excessive public deficit that induced serious debt problems. Figure I also plots, between 1977 and 2016, one of the two official measures that the Ministry of Finance (SHCP) computes regarding fiscal deficit relative to GDP called *Balance Público Tradicional* (BPT). This measure represents the difference between current and capital expenditures with total revenue of the public sector, although, it does not consider the revenue/expenditures of *Banco de México* neither the public financial sector. Consequently, this deficit measure represents the primary deficit plus the financial cost of debt of the non-financial public sector. It is worth mentioning that the BPT series that I present between 1977-1989 is an historical construction made by *Banco de México* using the limited available data on the primary deficit and financial cost of debt during these years.

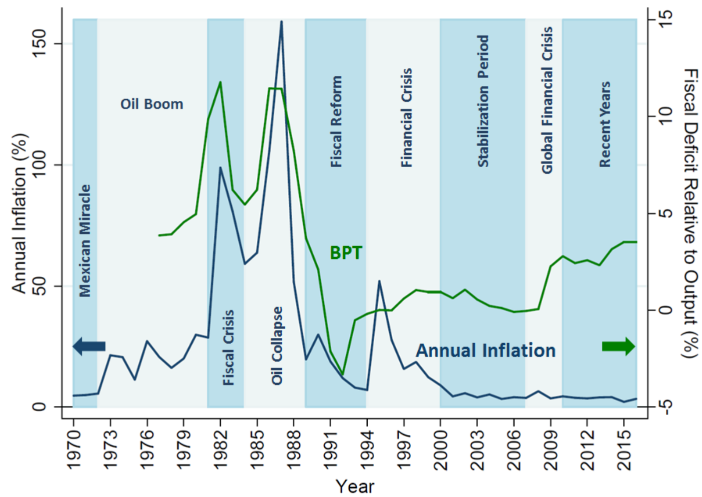
According to the figure, there is an important correlation between the annual inflation rate and fiscal deficit relative to output especially during those years in which inflation was high. To understand the circumstances that led to high inflation episodes in Mexico during 1960 and 2016, this part of the paper presents a review of the fiscal and inflation history between these years. This section is mainly inspired in Cárdenas (2015) among other studies this author has. Therefore, unless it is mentioned on a footnote, I am presenting historical facts analyzed by this author.

3.1 1960s: The Mexican Miracle

During 1960 and 1971, Mexico experienced an important economic growth phase known as **The Mexican Miracle**. Between these years, real output had an average annual growth rate of 7.13%

13 The INPC is Mexico's Consumer Price Index (CPI) that is computed by the National Institute of Statistics and Geography, *Instituto Nacional de Estadística y Geografía* (INEGI), since 2011. Before that year, Mexico's Central Bank (*Banco de México*) computed this index.

Figure I: Annual Inflation and Fiscal Deficit Relative to Output.



Source: Banco de México, INEGI, and SHCP.

Notes: this figure plots in blue the annual inflation rate computed with the INPC's historical data between 1969 and 2016. In green, the figure displays the BPT, an historical fiscal deficit measure. The displayed BPT before 1990 is a construction computed with the limited data on fiscal deficit between 1977-1989. The BPT series presents a methodology change in 1990, 2008, and 2014.

that represented a per-capita growth of 3.75%. This period is characterized by the lowest inflation of the second half of the twentieth century, with an average of 2.8%.[14]

This vast economic growth was primarily driven by the high investment rate of the private and public sector: private/public investment had an annual average growth of 11.5%/7%. Mexico experienced an important transformation during these years, especially the urban zones that had a considerable population growth and industrialization. The government provided services also had an important expansion, particularly educational and health services. This had an impact on poverty, that was reduced at the end of the 60s. Another consequence of this economic growth, was the boom presented in the financial sector. For example, between 1960 and 1971, the population that had access to credit went from 3% to 15% and loans as a percentage of GDP went from 15% in 1958 to 30% by 1970.

This phase of the Mexican economic history is characterized by an active government interventionism in some key sectors of the economy. The final objective of the policies adopted by the government was to reduce the foreign presence in the Mexican economy. Among this interventionist policies were: (i) controlling the price of commodities; (ii) an import substitution policy; (iii) restricted foreign capital in the private sector; and (iv) began to invest in firms that were near bankruptcy in order to save jobs and production. Evidently, public expenditures suffered an important growth during the Mexican miracle. On top of that, the Fiscal Authority had a lot of problems collecting taxes during these years. In fact, Mexico's financial sector was the government's main income source.

Cárdenas (2015) argues that, even though an economic crisis presented until 1982, many structural problems that eventually led to a crisis were originated during the Mexican miracle. The source of these problems was the excessive government interventionism in economic activity, that had as

14 All output growth and inflation rate data presented in this section were obtained in the *historical Mexican Statistics* published in INEGI (2014).

a consequence: (i) loss of competitiveness in the private sector;[15] (ii) an accelerated growth of groups with sufficient power to influence political decisions; (iii) reinforcement of the oligopolistic structure; (iv) low trade liberalization; and (v) an enlargement of fiscal deficit relative to output. These, among other problems, put an end to the Mexican miracle at the beginning of 1972.

3.2 1970s: Oil Boom

At the end of 1971, a global recession occurred and international credit was reduced. To avoid a stagnation period, president Luis Echeverria's administration decided to use public expenditure to drive the national economy. Given that the Federal Government had problems collecting taxes, these expenditures were financed through money creation, reserves that private banks had to make in the Central Bank, and foreign credit. As a consequence, fiscal deficit relative to output went from 2.5% in 1971 to 4.9% by 1972. Also, the monetary base grew 14.8% during 1972.[16] Even though during 1972 the annual inflation rate was low (2.6%), the poor financial management of the public policy and the monetary emission had as a consequence an average inflation of 14% during 1973-1976.

The fiscal and monetary expansions gave results during 1973: real output had a growth rate of 8.4% that represented a per capita growth of 4.9%. However, fiscal expenditures expanded 23.2% during this year. Echeverría's administration knew that it had to increase their incomes to reduce fiscal deficit, but hardly any fiscal reform was made to do this. By 1976, the deficits generated since 1973 induced several debt problems. Output growth during 1976 was 4.2% and inflation was 27.2%, suggesting that the government's policy to invest and drive the national economy had failed.

In 1977 an important oil discovery (in Cantarell, Campeche) allowed the Federal Government to renegotiate all the debt it had. Actually, Mexico's foreign credit increased. This oil boom induced an average output growth rate of 7.8% and an inflation rate of 24.2% between 1977-1981. Still, this accelerated growth was driven, once again, by government expenditure that went from 30.9% (as a proportion of output) to 40.6% by 1981. Meanwhile, public revenue increased because of the oil sales although not as fast as expenditures. Consequently, fiscal deficit relative to output went from 6.7% in 1977 to 14.1% by 1982.

3.3 1981-1987: Oil and Fiscal Crisis

In 1981 the world experienced another recession that reduced once again international credit. To prevent a higher deficit in the balance of payments, president José Lopez-Portillo decided to maintain the exchange rate fixed and to control imports. Despite this potential crisis, no serious reform was made to change public expenditures. As a consequence, by 1982, debt payment became infeasible and the international reserves held by *Banco de México* were depleted. Given this fiscal crisis, several credits that were approved for Mexico were cancelled. This had as a consequence a severe capital outflow. To handle this situation, president Lopez-Portillo decided to nationalize all private banks in order to control capital outflow and the exchange rate.

The lack of foreign credit forced the government to finance most of its expenditures with money creation: between 1981 and 1983, the monetary base had an annual average growth rate of 90.4%. Consequently, inflation during this period was on average 63.1%. Given that inflation was high and economic growth was stagnant, between 1983-1986 the government had no choice but to increase taxes and renegotiate foreign credits (with a considerably high interest rate) to reduce its deficit.

15 This loss of competitiveness was primarily caused by two sources: artificial prices of commodities and the rescue of several firms that weren't allowed to be declared in bankruptcy.

16 All the monetary base growth rates were computed with *Banco de México*'s data on this variable.

However, a fundamental reform to change public expenditure was not made. Thus, even though during 1983-1984 fiscal deficit was reduced, by 1986 it reached the same level as before: 14.1% of GDP.

On top of this, during 1985 oil prices began to drop at a global scale. By 1986 the Mexican oil price had a drop of 65%. This oil crisis generated a loss of 8,500 million dollars in sales (6.5% of the GDP) and a reduction of 26% in the federal income. With this "crisis within the crisis" president Miguel de la Madrid decided to protect the balance of payments, international reserves and employment by sacrificing inflation: by 1987 the annual inflation rate was 159%.

3.4 1988-1995: Fiscal Reform and Financial Crisis

In 1988 the government and representatives of the private sector made an agreement called Economic Solidarity Plan (*Pacto de Solidaridad Económica*) in which the government agreed to reduce its expenditures and inflation using the exchange rate as nominal anchor. Additionally, president Carlos Salinas de Gortari impulsed several fiscal reforms and renegotiated foreign debt. This fundamental reform was successful: by 1989 the annual inflation rate was 20.3%. During president Salinas de Gortari's period (1988-1994), the public deficit presented historical minimums and even achieved surplus.

With the beginning of the North America Free Trade Agreement (NAFTA) and the re-privatization of private banking made in 1994, Mexico had high development expectations. However, several factors induced another crisis at the end of this year and during 1995. One of these factors was the re-privatization of private banking. The privatization process generated incentives for the new owners to issue foreign debt in order to buy a bank. This left the private financial sector exposed to sudden exchange or interest rate movements. On the other hand, the government issued a considerable amount of bonds whose nominal value was in dollars but were paid in pesos (called *Tesobonos*). Because a lot of these bonds were accepted by the market, the government needed to maintain a stable exchange rate, otherwise its debt would have exploded. However, because of certain political events, at the end of 1994 there was an important depreciation and a considerable capital outflow.[17]

As a consequence, the government had a potential debt crisis and the private financial sector was in bankruptcy. Inflation during 1995 was 51%. The Mexican government negotiated with the International Monetary Fund (IMF) and with the U.S. several credits to pay most of its debt. Meanwhile, the private financial sector received several credits from *Banco de México* and resources from a trust fund called *Fondo Bancario de Protección al Ahorro* (FOBAPROA).

3.5 1995-2016: Stabilization

Considering the consequences that fiscal dominance had on inflation during the 80s, in 1993 the Mexican constitution had a reform through which *Banco de México* became an autonomous institution. This means that the Central Bank cannot finance public deficit with money creation (or any other instrument). Hence, it is now the Fiscal Authority that has to adjust its deficit to prevent a debt crisis. In April 1994, *Banco de México* became fully independent having as its primary constitutional mandate to control the purchasing power of the national currency.

Some of the policies adopted after 1994 by this Central Bank were: (i) restore the international reserves to gain credibility; (ii) began to use an objective for the cumulative current account balances that private banks deposit on the Central Bank as its monetary policy instrument (called

17 Calvo and Mendoza (1996), Cole and Kehoe (1996), and Sachs et al. (1995) analyze the causes and consequences of this crisis.

corto); (iii) adopted an inflation-targeting policy that aims an annual inflation of 3% that can float between 2% and 4%; and (iv) to be more transparent, it started publishing documents to explain the monetary policy decisions among a quarterly report about the Mexican economy.[18]

As a result of these policies, the average annual inflation rate went from 10.95% between 1996-2002 to 3.98% between 2003-2016 achieving historical minimums in 2016. Also, as documented by Chiquiar et al. (2007), the inflation rate after 2002 became a stationary process that started converging to *Banco de México*'s inflation target.

Meanwhile, fiscal deficit has remained relatively low and controlled between 1997-2016. In order to follow international standards (e.g., those of the IMF), the BPT had a methodology change in 2008 and 2014 that made it a wider fiscal deficit measure.[19] This could explain the fiscal deficit rise observed in Figure I after 2008.

4 The Model

In this section I present a model that explains the evolution of Mexico's inflation rate between 1969 and 2016 as a result of inflationary expectations and an exogenous fiscal deficit sequence. Hence, the model allows me to analyze the interaction between these variables during the past 48 years in Mexico.

I replicate the framework presented in Sargent et al. (2009), a paper that studies the relationship between observed inflation, inflationary expectations, and fiscal deficits in several Latin American countries. Given its relatively simple structure, one advantage of the model presented in this paper is that it generates a sequence for each of the relevant variables (inflation rate, its expectations, and fiscal deficits) only from historical data on one of these variables. Therefore, extended periods of a country's inflation and fiscal history may be analyzed without needing data on all the variables involved in the model. In the case of Mexico and several Latin American countries, this methodology is ideal due to several complications with the fiscal and inflationary expectations data. In this paper, I consider the sequence of monthly gross inflation computed with the *Índice Nacional de Precios al Consumidor* (INPC) between January 1969 and December 2016. From this historical inflation the model estimates the sequence of inflationary expectations and fiscal deficits that better accounts for the inflation rate presented in the data conditional on the model's structure.

4.1 Set up

There are two fundamental equations that set up the model, inspired in the framework proposed by Cagan (1956). These equations relate, (i) the nominal balances as a percentage of output at time t (M_t); (ii) the price level at time t (P_t); (iii) agents' expectations about the price level in $t+1$ (P^e_{t+1}); and (iv) fiscal deficit as a percentage of output (d_t); as follows:

$$\frac{M_t}{P_t} = \frac{1}{\gamma} - \frac{\lambda P^e_{t+1}}{\gamma P_t}, \tag{1}$$

$$M_t = \theta M_{t-1} + P_t d_t, \tag{2}$$

18 For more on the policies adopted by *Banco de México* to reduce inflation after 1995, consult Ramos-Francia and Torres-Garcia (2005).

19 After 2008, the BPT considers part of the investments made by two important state-owned firms (*PEMEX* and *CFE*) that before were considered as long-termn debt (investments of this type are called *PIDIREGAS*). Hence, before 2008 these expenditures were not part of the BPT neither other fiscal deficit measures.

where: $0 < \lambda < 1$ represents the weight that agents' expectations have on P_t,[20] $\gamma > 0$ is the weight that the nominal balances relative to output have on the price level, and $0 < \theta < 1$ represents the persistence of the nominal balances between two consecutive periods.[21]

Equation (1) represents a functional form for the economy's real balances demand. Throughout this paper I take this equation as given, however, there are some papers (like Marimon and Sunder (1993)) that develop micro-founded models in which equation (1) can be derived from an optimization framework.[22] According to this equation, if the public expects a price level increase during $t+1$, their real balances demand (M_t/P_t) will fall.

On the other hand, equation (2) represents the part of the fiscal deficit that must be financed with monetary emission, thus, it considers the deficit net of any debt emissions.[23] According to equation (2), if the government enlarges its deficit relative to output (d_t) then the nominal balances as a percentage of GDP (M_t) should also increase. The sequence $(d_t)_{t=0}^T$ is considered exogenous (i.e., inflation does not affect the fiscal policy decisions that induce this sequence). Consequently, this model is explicitly assuming that the economy is in a permanent fiscal dominance regime since $(d_t)_{t=0}^T$ is always financed with money creation and it is considered exogenous.[24]

If the public's gross expected inflation rate is defined as $\beta_t = P_{t+1}^e/P_t$, using (1) and (2), it can be shown that the gross inflation rate at time t is:

$$\pi_t = \frac{P_t}{P_{t-1}} = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t - \gamma d_t}. \tag{3}$$

This equation suggests that inflation is a function of just two variables: agents' inflationary expectations and the government's fiscal deficit relative to output. According to (3), if the expected gross inflation (β_t) or the fiscal deficit (d_t) rises, inflation (π_t) will also go up.[25] It is worth mentioning that the equation that governs inflation is independent of any assumption made about the evolution of expectations ($\beta_0, \beta_1, \ldots$) or for the process that determines the fiscal deficit sequence ($d_0, d_1, \ldots$). Nevertheless, these sequences are crucial to determine a sequence of inflation ($\pi_0, \pi_1, \ldots$) according to the model. Hence, the next two sections will explain all the assumptions made regarding the evolution of expectations and the dynamics that govern fiscal deficits.

4.2 Expected Inflation Rate Evolution

In this model, also inspired by Cagan (1956), it is assumed that agents update their beliefs on future inflation (β_t) using adaptive expectations. According to Sargent and Wallace (1973), agents

20 Equation (1) can be written as $P_t = \gamma M_t + \lambda P_{t+1}^e$. Hence $\{\lambda, \gamma\}$ represent the weight that P_{t+1}^e and M_t have respectively on P_t.

21 The parameter θ is also related to output growth in the model. Let $\hat{M}_t = M_t/Y_t$ where $\hat{M}_t$ represents the nominal balances at time t and Y_t is output. Considering D_t as the fiscal deficit at time t then the government budget constraint is: $\hat{M}_t = \hat{M}_{t-1} + P_t D_t$. Dividing this equation by Y_t then: $M_t = (Y_{t-1}/Y_t)M_{t-1} + P_t d_t$. Therefore, θ can be interpreted as the inverse of the output growth factor.

22 In Marimon and Sunder (1993), the authors develop an overlapping generations model where agents live for two periods, one consumption good (money), and uncertainty about the endowments of future agents. As a result of the expected-utility maximization of each agent, they show that agents demand real balances according to (1).

23 Equation (2) is a usual government constraint. The monetary and fiscal constraint of a consolidated government is: $g_t - \tau_t = (M_t - \theta M_{t-1})/P_t + b_{t+1} - (1+r_t)b_t$, where g_t, τ_t represents the government expenditures/revenue relative to output during t, b_t is the debt that the government has at time t relative to output, and r_t is the interest rate that the government pays for that debt. Defining fiscal deficit as the primary deficit plus the net debt emissions ($d_t = g_t - \tau_t + (1+r_t)b_t - b_{t+1}$ for all t) this constraint becomes (2).

24 In Section 6.4 I discuss the interpretation that the model gives to *Banco de México*'s autonomy, which was granted in 1994.

25 For every $0 < \lambda, \theta < 1$, and $\gamma > 0$, it can be easily shown that: $\partial\pi_t/\partial\beta_t > 0$ and $\partial\pi_t/\partial d_t > 0$.

have adaptive beliefs when they only consider past information and extrapolate from it to form their expectations. Particularly, this model assumes that the public's gross expected inflation rate is a weighted average between the gross inflation rate and the gross expected inflation lagged one period:

$$\beta_{t+1} = (1-\nu)\beta_t + \nu\pi_t, \tag{4}$$

where $0 < \nu < 1$ is the weight that agents give on their beliefs to the past observed inflation. In the literature, this class of adaptive expectations are known as **Constant-Gain Expectations** (CGE) since agents always consider the same weight on the variables they use to form their expectations.[26]

The CGE assumption has several repercussions on inflation dynamics. One important consequence is the relationship implied by CGE for inflation in two consecutive periods (π_{t+1} and π_t). Figure II shows the relationship between the difference $\pi_{t+1} - \pi_t$ and β_t considering the same fiscal deficit level in both periods. As shown in the figure, CGE implies a quadratic relationship between $\pi_{t+1} - \pi_t$ and β_t.[27] Consequently, there are two values of β that imply a constant inflation over time: β_1 and β_2. In the adaptive expectations literature, β_1 and β_2 are known as **Self-Confirming Equilibria**. As implied by the figure, β_1 is a (local) stable equilibrium, thus, if the public's belief on future inflation is not considerably high then $\pi_{t+1} - \pi_t$ will converge to zero and expectations to β_1. Additionally, equation (4) implies that inflation will also converge to β_1. On the other hand, if $\beta_t > \beta_2$ then the difference $\pi_{t+1} - \pi_t$ will begin to grow and these dynamics are unbounded. Therefore, $\beta_t > \beta_2$ implies that the model will eventually generate an hyperinflation episode. Sargent et al. (2009) define this phenomena as an **Escape Dynamics**.[28]

Panel **(b)** of Figure II presents another consequence of CGE: assuming β_t induces an escape, if the government reduces its deficit permanently then it can prevent an hyperinflation episode. This figure shows two dynamics for $\pi_{t+1} - \pi_t$ as a function of β_t. The only difference between both curves is the fiscal deficit level: the dynamics shown in blue are related to a high deficit while the dynamics in green are a consequence of a low deficit. Assuming that the government produces a high deficit and $\beta_t = \hat{\beta}$, if the government continues with a similar deficit then it will provoke an escape dynamics as shown with blue arrows in panel **(b)** of Figure II. On the other hand, if the government permanently reduces its deficit then, even though $\beta_t = \hat{\beta}$, it will be able to arrest inflation and prevent an escape. Actually, $\pi_{t+1} - \pi_t$ will converge to a low and stable inflation equilibrium as shown by the green arrows in the figure.

Finally, one important implication of CGE in this framework is computational: assuming these type of expectations allows to compute them easily. Therefore, the complexity of the function that will be used to estimate all the parameters is substantially reduced (Section 5 explains all the estimation details). Despite these implications of CGE, there are many studies about hyperinflation episodes that consider another type of expectations, called rational expectations, that also adjust well to the data. The main implications of rational expectations (in the context of this model) and its differences with an adaptive scheme are detailed in Appendix A.

26 For more on constant-gain expectations see, for example, Marcet and Nicolini (2003) and Malmendier and Nagel (2015).

27 Given β_{t-1} and assuming a constant fiscal deficit (equal to $\bar{d}$) then:

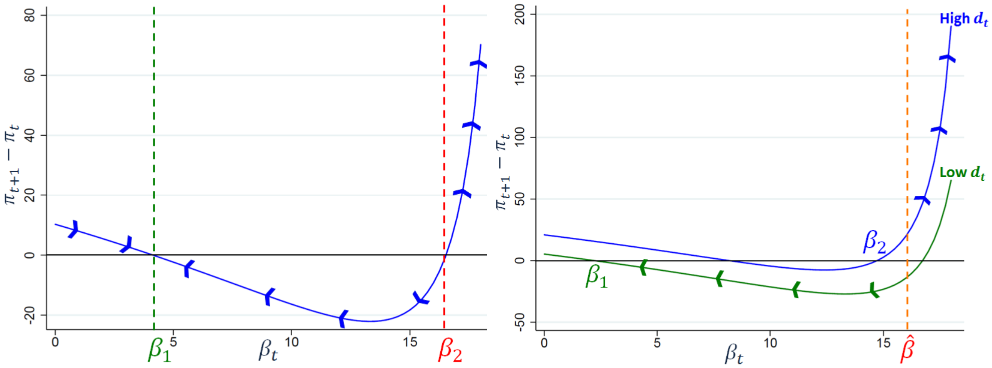
$$\pi_{t+1} - \pi_t = \frac{\theta(1-\lambda\beta_t)}{1-\lambda\beta_{t+1}-\gamma\bar{d}} - \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t-\gamma\bar{d}}.$$

CGE implies that β_{t+1} is a function of β_t. Hence, for relatively high values of β_t, $\partial\pi_{t+1}/\partial\beta_t > 0$. And since for all β_t, $\partial\pi_t/\partial\beta_t > 0$, this implies that $\pi_{t+1} - \pi_t$ is quadratic in β_t.

28 Williams (2016) characterize how adaptive expectations could lead to an escape dynamics and explains how the likelihood, frequency and direction of the variables during an escape dynamics are always characterized by a deterministic control problem.

Figure II: Dynamics Induced By Adaptive Expectations.

(a) Inflation and Expectations. (b) Fiscal Deficit and Expectations.



Notes: these figures consider $\beta_{t-1} = 1.02$ and the estimated parameters with Mexican data shown in Table II.

4.3 Fiscal Deficit Relative to Output Dynamics

The other crucial variable that determines the gross inflation rate (π_t) is fiscal deficit relative to output (d_t). Considering that fiscal policy may be influenced by exogenous conditions like the state of the global financial markets, the international price of commodities that are crucial to determine the government's income (e.g., oil), or even political conditions, in this model (although it is a simplified way to capture all of the exogenous conditions that affect the Mexican economy) it is assumed that d_t is a random variable with the following conditional distribution:

$$log(d_t|\bar{d}_t, v_t) \sim N(log(\bar{d}_t), v_t). \tag{5}$$

Thus, $d_t|\bar{d}_t, v_t$ is a random variable with a log-normal distribution that has a median of $\bar{d}_t$ and a variance parameter v_t. Each period $\bar{d}_t$ is determined by a discrete Markov process with D possible states.[29] Similarly, v_t follows another Markov process with V states that is independent of the process that determines $\bar{d}_t$.[30] In the literature, the stochastic process followed by d_t is known as a **Hidden Markov Process**.[31] The parameteres $\{\bar{d}, v\}$ are the fiscal deficit hidden states, which are unobservable to agents and econometricians. Each Markov process is related to a matrix whose elements represent the transition probabilities from one state of the process to another. Let $Q_d \in \mathbb{R}^{D\times D}, Q_v \in \mathbb{R}^{V\times V}$ be the transition matrix associated to the $\bar{d}_t, v_t$ process respectively.[32]

Assuming that fiscal deficits are governed by a hidden Markov process allows the model to form a non-linear relationship between inflation, expectations, and public deficit. This occurs since the effect of current inflationary expectations (β_t) on inflation (π_t) and future expectations (β_{t+1}) is

29 A stochastic sequence $(x_t)_{t=0}^{\infty}$ is said to be a discrete Markov process if x_t takes values in a set I with $|I| \in \mathbb{N}$, and for all $t = 1, 2, \ldots$ the Markov property is satisfied: $P[x_{t+1} = i|x_0, x_1, \ldots, x_t] = P[x_{t+1} = i|x_t]$. This property states that, in a Markov process, past realizations of x $(x_0, x_1, \ldots, x_{t-1})$ do not affect future values. Only the current state (x_t) affects x_{t+1}.

30 For every $\bar{d}, v$, and t : $P[\bar{d}_t = \bar{d}, v_t = v|\bar{d}_{t-1}, v_{t-1}] = P[\bar{d}_t = \bar{d}|\bar{d}_{t-1}]P[v_t = v|v_{t-1}]$.

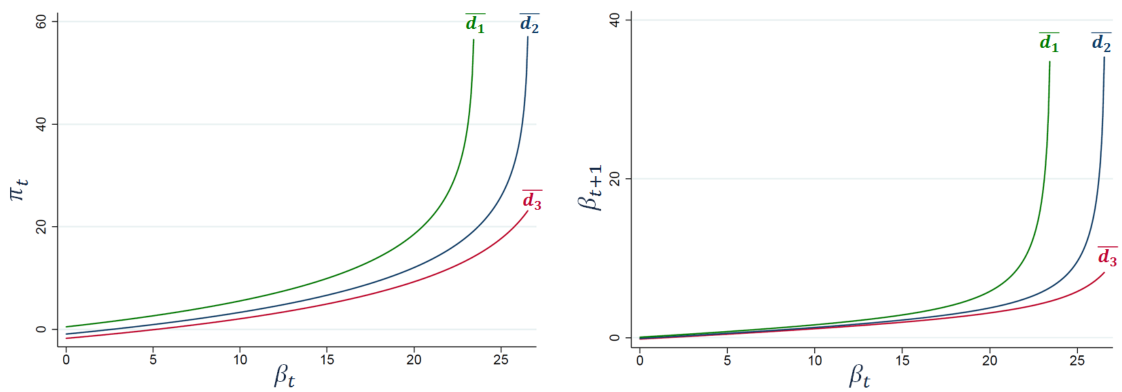
31 A hidden Markov process is a pair $(x_t, y_t)_{t=0}^{\infty}$ such that: x_t is a (usual) Markov process, there exists a function f such that $y_t = f(x_t)$, and for all t: $P[y_{t+1} = y|x_0, x_1, \ldots, x_{t+1}, y_0, y_1, \ldots, y_t] = P[y_{t+1} = y|x_{t+1}]$. In processes of these type, y_t is known as the observable part, and x_t is the hidden component. In the model presented in this paper, y_t is the fiscal deficit relative to output meanwhile x_t is a vector whose elements are the median $\bar{d}_t$ and variance v_t parameters. All the methodology that I follow regarding hidden Markov processes is based on Sims et al. (2006) and MacDonald and Zuccini (2009).

32 For example, Q_d in its $i, j-$component contains the probability of being in a state j in $t + 1$ conditional on $d_t = i$, i.e., $Q_d(i, j) = P[\bar{d}_{t+1} = j|\bar{d}_t = i]$.

a function of the hidden Markov state that governs median fiscal deficit ($\bar{d}_t$).[33] An example of this non-linear relationship within the variables of the model is presented in panels **(a)** and **(b)** of Figure III. Panel **(a)** shows that the effect of β_t on inflation magnifies as the hidden state $\bar{d}_t$ rises (this figure considers $\bar{d}_1 > \bar{d}_2 > \bar{d}_3$). Panel **(b)** displays a similar influence on the inflationary expectations dynamics.

Figure III: Non-Linear Effect of Fiscal Deficit on the Model.

(a) Effect on Inflation. (b) Effect on Expectations.



Notes: these figures consider $\bar{d}_1 > \bar{d}_2 > \bar{d}_3 > 0$, $\beta_{t-1} = 1.02$, and the estimated parameters with Mexican data shown in Table II.

This non-linear relationship between inflation, expectations, and fiscal deficits within the model is consistent with several empirical studies about the fiscal policy effect on inflation. For example, Catao and Terrones (2005) and Lin and Chu (2013) provide evidence (considering approximately 100 countries) that fiscal deficit has a strong/weak impact on inflation during high/low inflation episodes. In this sense, a fiscal consolidation/imbalance will only have an important effect stabilizing/raising prices the higher inflation is. Therefore, according to this model and the data, when inflation is controlled the fiscal policy has a limited effect on this variable.

Finally, one consequence of assuming a log-normal distribution for fiscal deficit relative to output is that d_t cannot be negative. Hence, the model does not allow a fiscal surplus. Sargent et al. (2009) explain that even when they changed the distribution of d_t allowing it to be negative, the model's fit to the data did not improve. Besides, they argue that a log-normal distribution captures the skewness of inflation shown in the data. Appendix B shows that a log-normal distribution for d_t implies a similar inflation density.

4.4 Model Constraints

As equation (3) implies, $\pi_t = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t-\gamma d_t}$, inflation in the model is well defined only if at each t: $1 - \lambda\beta_{t-1} > 0$ and $1 - \lambda\beta_t - \gamma d_t > 0$ (otherwise the real balances demand could become negative). However, the model does not consider any endogenous restriction that prevents one of these constraints from being violated. Additionally, (3) implies that the gross inflation rate is not bounded: if $1 - \lambda\beta_t - \gamma d_t \to 0$, then $\pi_t \to \infty$. Given the numerical problem that this property causes in the

33 The fiscal deficit can be decomposed as $d_t = \bar{d}_t + \epsilon_t(v_t)$ where $\epsilon_t(v_t)$ is a log-normal distributed shock (with parameters $\mu = 0$ and $\sigma^2 = v_t$). In the absence of this shock, the model's inflation rate is $\pi_t = \theta(1 - \lambda\beta_{t-1})/(1 - \lambda\beta_t - \bar{d}_t)$. Because β_t and $\bar{d}_t$ are in the denominator of π_t, as the median deficit hidden state enlarges, the effect of expectations on inflation will grow exponentially.

parameter estimation, it is assumed that there exists a constant $\delta > 0$ such that $\pi_t < \delta$ for every t. Then again, there is no restriction within the model that prevents $\pi_t \geq \delta$ for some t. Appendix B provides further insight about the numerical problem that $\pi_t \geq \delta$ could cause when estimating the model's parameters.

The two restrictions that need to be considered such that π_t is well defined and bounded are:

$$1 - \lambda\beta_{t-1} > 0 \;\text{ and }\; \delta(1 - \lambda\beta_t - \gamma d_t) > \theta(1 - \lambda\beta_{t-1}). \tag{6}$$

If any of these constraints is violated then it is assumed that the gross inflation rate is not given by (3). Instead, π_t will be randomly determined according to the following log-normal distribution:

$$log(\pi_t) \sim N(log(\bar{\pi}_t(d_t)), v_\pi), \tag{7}$$

where $\bar{\pi}_t(d_t)$ is the inflation equilibrium determined by (3) in the model without uncertainty and conditional to a certain fiscal deficit d_t.[34] Additionally, if $\delta(1 - \lambda\beta_t - \gamma d_t) \leq \theta(1 - \lambda\beta_{t-1})$, Sargent et al. (2009) suggest to reset expected inflation to $\beta_{t+1} = \pi_t$. Otherwise, the dynamics between β_{t+1} and inflation will provoke $\pi_{t+1} \geq \delta$ and eventually $\beta, \pi \to \infty$.

Whenever the current hidden Markov state $\{\bar{d}_t, v_t\}$ induces certain dynamics that will eventually make $\{\pi, \beta\}$ violate (6) or that will generate an escape event, the government can implement a **Reform** to prevent this from happening. Sargent et al. (2009) define two types of reforms: a reform is said to be **Cosmetic** if the government controls inflation but the fiscal deficit level is not altered or it has a temporal reduction. A cosmetic reform occurs when the hidden Markov state $\{\bar{d}_t, v_t\}$ changes temporarily or when inflation is randomly determined.

On the other hand, a **Structural** reform occurs whenever the government controls inflation but also reduces its deficit level more permanently. Following panel **(a)** of Figure II, a cosmetic reform could fail if the expected inflation rate β_{t+1} is greater than β_2. However, a cosmetic reform can be successful if $\beta_{t+1} \leq \beta_2$. For example, Sargent et al. (2009) argue that in Peru a cosmetic reform was enough to control the inflationary crisis that this country had during the 80s. Panel **(b)** of Figure II is an example of a structural reform in which the government successfully controls inflation and expectations.

One important contribution of Sargent et al. (2009) to the literature is that its model identifies if a reform is cosmetic or structural. Previous studies considered only structural reforms that could arrest high inflation episodes, even though the notion of a cosmetic reform was part of the discussions (for example, in Marcet and Nicolini (2003)). Besides, Sargent et al. (2009) justify including cosmetic reforms in their model since "they are crude devices designed to mimic various things that Latin American governments did in the 80s to arrest inflation "on the cheap" without tackling fiscal deficits".

Consequently, following (3), (7) and defining $\mathcal{X}_t$ as follows, the gross inflation rate π_t is formally determined as:[35]

$$\pi_t = \mathcal{X}_t \frac{\theta(1 - \lambda\beta_{t-1})}{1 - \lambda\beta_t - \gamma d_t} + (1 - \mathcal{X}_t)\, \pi_t^\star(d_t), \text{ with} \tag{8}$$

$$\mathcal{X}_t = \mathcal{X}_{\{1-\lambda\beta_{t-1}>0\}} \mathcal{X}_{\{\delta(1-\lambda\beta_t-\gamma d_t)>\theta(1-\lambda\beta_{t-1})\}},$$

34 Certainty in the model implies $\pi_t = \beta_t$. In equilibrium, $\pi_t = \pi_{t-1}$. Using (3) it can be shown that:

$$\bar{\pi}_t(d_t) = \frac{1 + \theta\lambda - d_t - \sqrt{(1 + \theta\lambda - d_t)^2 - 4\theta\lambda}}{2\lambda}.$$

35 $\mathcal{X}_t$ is a constant equal to 1 if $\{\pi_t, \beta_t.d_t\}$ satisfy the constraints shown in (6) and is zero otherwise.

where: $\mathcal{X}_A$ is the characteristic function of the set A, and $\pi_t^\star(d_t)$ is a simulated inflation value according to (7).

5 Parameter Estimation

The following equations (together with the transition matrices Q_d, Q_v) define inflation, expected inflation, and fiscal deficit relative to output at each t according to the model:

$$\pi_t = \mathcal{X}_t \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t - \gamma d_t} + (1-\mathcal{X}_t)\,\pi_t^\star(d_t),$$

$$\beta_{t+1} = (1-\nu)\beta_t + \nu\pi_t, \quad log(d_t|\bar{d}_t, v_t) \sim N(log(\bar{d}_t), v_t).$$

Assuming $\beta_0 = \pi_0$, these equations define a sequence $(\pi_t, \beta_t, d_t)_{t=1}^T$ that can be interpreted as the historical relationship between these variables. However, to construct these sequences, parameters like $\{\lambda, \gamma, \delta, \theta\}$ among the hidden Markov states $\{\bar{d}, v\}$ must be calibrated or estimated. This section explains the procedure I follow to calibrate or estimate all the parameters involved in the model.

5.1 Block-Wise Maximum Likelihood Method

Table I shows all the parameters involved in the model. Since d_t is a random variable and $\{\pi_t, \beta_t\}$ are a function of fiscal deficit, I could construct a joint density function for a T period inflation, expectations, and fiscal deficits sequence, $p(\pi^T, \beta^T, d^T|\phi)$, where ϕ is the vector of all the parameters involved in the model. If there were available data on inflation, expectations, and fiscal deficits for a large T then the parameters could be estimated using the maximum-likelihood method applied to the joint density $p(\pi^T, \beta^T, d^T|\phi)$. However, as Sargent et al. (2009) suggest, data on these variables is hard to find for a large T. These authors even argue that data on fiscal deficit is not reliable for several countries this paper studied. In Mexico's case, there are additional complications related with the fiscal deficit data: (i) there are several official definitions of this variable (some measurements include the public financial sector and others do not); (ii) the historical sequence before 1990 was not computed at that time, it is a construction made with the limited data on primary deficit and the financial cost of debt during those years; (iii) since the perception of what constitutes fiscal responsibility for the government has evolved in recent years, there have been several methodology changes to make the fiscal deficit measurement wider in order to follow international standards (e.g., those of the International Monetary Fund).[36]

Nevertheless, Mexico does have reliable data on inflation for a large T. The historical INPC sequence is available since January 1969 at a monthly frequency. Therefore, to estimate ϕ I consider the marginal density of the inflation sequence π^T between January 1969 and December 2016. This marginal density is denoted $p(\pi^T|\phi)$ and the procedure that I follow to compute this density is detailed in Appendix B.

Considering the gross inflation rate sequence π^T, the vector of estimated parameters ($\hat{\phi}$) is the one that maximises $p(\pi^T|\phi)$ given the inflation data and subject to the constraints that are shown in Table I:[37]

$$\hat{\phi} \in \text{argmax}_{\phi\in\Omega} \;\; p(\pi^T|\phi). \tag{9}$$

36 For example, after the 1995 crisis, the government started to consider additional liabilities related to the financial public sector, such as overdue portfolios and loans. Another example is that, since 1998, the government computes the financial requirements of a particular type of state-owned firms' long-term expenditures, called PIDIREGAS (*Proyectos de Inversión Pública Financiados por el Sector Privado*). However, these type of expenditures were not explicitly considered part of the BPT (or neither other fiscal deficit measures) until 2008.

37 Ω is the set of all the vectors ϕ that satisfy the constraints shown in Table I.

Table I: Model Parameters.

Parameter	Restriction	Interpretation
λ	$0 < \lambda < 1$	Weight of expectations on the price level
γ	$\gamma > 0$	Weight of monetary base on the price level
θ	$0 < \theta < 1$	Persistence of the monetary base
ν	$0 < \nu < 1$	Weight of past inflation on expectations
δ	$\delta > 0$	Constant that bounds inflation
$\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_D$	$\bar{d}_1 > \bar{d}_2 > \ldots > \bar{d}_D > 0$	Fiscal deficit median
$v_1, v_2, \ldots, v_V$	$v_1 > v_2 > \ldots > v_V > 0$	Fiscal deficit variance parameter
v_π	$v_\pi > 0$	Inflation variance when determined randomly
p^d_{ij}	$0 \le p^d_{i,j} \le 1,\ \sum_j p^d_{i,j} = 1$	i,j-component of the transition matrix Q_d
p^v_{ij}	$0 \le p^v_{i,j} \le 1,\ \sum_j p^v_{i,j} = 1$	i,j-component of the transition matrix Q_v

Since there is no analytical solution to this maximization problem, $\hat{\phi}$ has to be numerically approximated. To do this, I consider the following block-wise constrained optimization algorithm based on Nocedal and Wright (2006) and Sims et al. (2006):

1. Given the complexity of the problem, Sims et al. (2006) suggest to partition the parameter set into blocks $\phi = [B_1, B_2, \ldots, B_n]$.

2. Taking the rest of the parameter blocks as given, update B_i maximizing $p(\pi^T|\phi)$ considering the constraints that each block has. This maximization is done using the constrained interior point with BFGS update algorithm described in Nocedal and Wright (2006).[38]

3. Once all the blocks are updated $(B_1', B_2', \ldots, B_n')$, the new parameter vector is $\phi' = [B_1', B_2', \ldots, B_n']$.

4. This process is repeated until $||\phi - \phi'||_2 < \epsilon$. If $||\phi - \phi'||_2 \ge \epsilon$, consider $\phi = \phi'$ and repeat steps (1)-(3).

To solve this maximization problem I considered $\epsilon = 10^{-8}$ and four blocks.[39] The log-normal assumptions together with the long sample that I am considering induce a likelihood of inflation that is well shaped around its global peak. However, since there are local peaks in which this numerical algorithm could stall, I considered a grid of 175 starting points for ϕ. The maximum-likelihood estimator $\hat{\phi}$ is the vector ϕ that the block-wise constrained algorithm founds with the highest log-likelihood. The other solutions that this algorithm found have a much lower likelihood, suggesting they are local peaks.

5.2 Estimation Considerations

The maximum-likelihood problem that needs to be solved is computationally intensive. Consequently, Sargent et al. (2009) fixed three parameters to reduce some of the estimation's complexity:

$$\theta = 0.99, \quad \delta = 100, \quad \gamma = 1.$$

38 The BFGS algorithm is an iterative numerical optimization tool that is highly efficient to minimize a function $f(x)$ that is numerically complicated. The main advantage of this method is that it approximates the second order derivatives of f with a semi-positive definite matrix. This allows the method to rapidly converge to a solution (if it exists) and also have a decent approximation of the Hessian matrix evaluated at the solution.

39 The four blocks are: $B_1 = (\lambda, \nu)$, $B_2 = (\bar{d}_1, \ldots, \bar{d}_D)$, $B_3 = (v_1, \ldots, v_V, v_\pi)$, and $B_4 = (p^d_{11}, p^d_{12}, \ldots, p^d_{DD}, p^v_{11}, p^v_{12}, \ldots, p^v_{VV})$.

This value of θ is consistent with the nominal balances behaviour in the five countries these authors studied.[40] This parameter implies that the model's nominal balances are persistent. Fixing $\delta = 100$ implies that inflation cannot overpass $10,000\%$. To make a sensitivity analysis of the estimated parameters as a function of δ, I made the parameter estimation considering $\delta \in \{2, 5, 50, 100\}$. The parameter estimation does not substantially change when δ is modified. Thus, I consider the same δ as Sargent et al. (2009).

The parameter γ was fixed due to an identification problem: equation (3) suggests that the gross inflation rate π_t is a function of γd_t. However, there is no other equation involving γ and d_t separately. Consequently, the maximization algorithm adjusts γd_t (not γ or d_t separately) to maximize the likelihood of the model. Sargent et al. (2009) prove that this arbitrary normalization does not affect the parameter estimation since it only rescales the model's likelihood. Once d_t is estimated, γ will be renormalized such that the mean of the deficit sequence generated by the model matches the mean of some data on fiscal deficit that Mexico had during 1977 and 2016.

Furthermore, the transition matrices Q_d, Q_v represent $D(D-1) + V(V-1)$ parameters that must be estimated. Sims et al. (2006) suggest to consider the following transition matrix (using Q_d as an example):

$$Q_d = \begin{pmatrix} p_{11}^d & 1-p_{11}^d & 0 & 0 & \cdots & 0 & 0 \\ \frac{1-p_{22}^d}{2} & p_{22}^d & \frac{1-p_{22}^d}{2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1-p_{33}^d}{2} & p_{33}^d & \frac{1-p_{33}^d}{2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1-p_{DD}^d & p_{DD}^d \end{pmatrix}$$

This simplification reduces the parameters related to Q_d from being $D(D-1)$ to just D. Besides, this assumption allows the hidden states to have a smoother transition between them. Considering a similar matrix for Q_v, the parameters to estimate are now V, not $V(V-1)$.

Finally, before estimating the parameters, one must choose the number of hidden states $\{\bar{d}, v\}$ (D, V respectively). As D or V become larger, the model's fit to the data might improve. However, as D or V grow, the computational cost of the maximum-likelihood problem increases since the number of parameters to estimate enlarges. Therefore, Sargent et al. (2009) only estimated two models for each country they studied. They considered a model with $D = 3, V = 2$ and another one with $D = 2, V = 3$. Then, the Schwarz information criterion (SIC) selects the model that has a better fit to the data.[41]

5.3 Estimation Results

After estimating the two models proposed by Sargent et al. (2009) with Mexican inflation data, the SIC suggests that $D = 3$, $V = 2$ induce a model with a better fit. Table II shows the estimation results together with each estimator's standard error. Following MacDonald and Zuccini (2009), I considered the Hessian matrix of the maximum likelihood problem to compute each standard error. Since the likelihood of inflation is very well shaped around the optimum, I can estimate the model's parameters with much accuracy. Thereby, the standard error of each estimated parameter is small.

40 Sargent et al. (2009) discuss the predictive power of the model if θ varies. They conclude that if θ is near to 1, the model predicts similar levels of inflation and its expectations. Hence, following Sargent et al. (2009) I consider $\theta = 0.99$.

41 The SIC is a Bayesian selection criterion between two models (A, B). Let L_x, P_x, n_x be the log-likelihood, the number of parameters and the sample size of model $x \in \{A, B\}$ respectively. The SIC for model x is computed as: $SIC_x = log(n_x)P_x - 2L_x$. If $SIC_A < SIC_B$ then, model A is preferred.

Table II: Parameter Estimation.

Parameter	Estimation	Interpretation
λ	0.7556 (0.0022)	Weight of expectations on the price level
ν	0.1147 (0.0081)	Weight of past inflation on expectations
$\bar{d}_1$	0.0075 (0.0001)	High fiscal deficit median
$\bar{d}_2$	0.0039 (0.0004)	Moderate fiscal deficit median
$\bar{d}_3$	0.0023 (0.0002)	Low fiscal deficit median
v_1	0.0671 (0.0087)	High fiscal deficit variance
v_2	0.0295 (0.0012)	Low fiscal deficit variance
v_π	0.0753 (.0010)	Inflation variance when it is determined randomly
p^d_{11}	0.9731 (.0361)	Probability of $\bar{d}_{t+1} = \bar{d}_1$ conditional on $\bar{d}_t = \bar{d}_1$
p^d_{22}	0.9787 (.0390)	Probability of $\bar{d}_{t+1} = \bar{d}_2$ conditional on $\bar{d}_t = \bar{d}_2$
p^d_{33}	0.9924 (0.0056)	Probability of $\bar{d}_{t+1} = \bar{d}_3$ conditional on $\bar{d}_t = \bar{d}_3$
p^v_{11}	0.7493 (0.1072)	Probability of $v_{t+1} = v_1$ conditional on $v_t = v_1$
p^v_{22}	0.7789 (0.0879)	Probability of $v_{t+1} = v_2$ conditional on $v_t = v_2$

Notes: these parameters were estimated using the algorithm described in Section 5.1. The numbers shown in parenthesis represent the standard error of each parameter. This standard error was computed using the Hessian matrix of the maximum likelihood problem, as suggested by MacDonald and Zuccini (2009). Since I am using monthly data from the INPC to construct the inflation rate, it is worth mentioning that all the estimation results have a monthly frequency as well.

The estimated parameters suggest the following facts about the price formation process in Mexico: $\lambda = 0.7556$ implies that agents' expectations have a considerable impact on the price level. This estimated λ suggests that inflation increases 0.56 p.p. given a 1% expectations rise.[42] Therefore, if inflationary expectations are volatile then the observed inflation will have a high variance as well. This result implies that a necessary condition to have a stable price level and inflation is to anchor expectations. Mexico's λ is similar to Sargent et al. (2009)'s estimation for Argentina ($\lambda = 0.730$), Peru ($\lambda = 0.740$), and Brazil ($\lambda = 0.875$).

Compared to other Latin American countries, $\nu = 0.1147$ implies that past observed inflation has a significant weight in agents' inflationary expectations. Sargent et al. (2009)'s estimation for Argentina ($\nu = 0.023$), Chile ($\nu = 0.025$), and Peru ($\nu = 0.069$) indicate that, in these countries, past inflation has a limited effect on expectations. Mexico's estimated ν suggests that observed inflation must remain stable for several months in order to anchor expectations.[43] On the other hand, this also implies that the expected inflation rate de-anchors only if observed inflation is high for a prolonged period.

Regarding fiscal deficits, the estimation suggests that: when the government generates a high fiscal deficit during one year ($\bar{d} = \bar{d}_1$ for twelve consecutive months) it represents approximately 9.12% of GDP. If the Mexican government produces a moderate deficit during one year then it is approximately 4.76% of GDP. Finally, if fiscal deficit is low during one year then it represents 2.78% of GDP. As will be shown in the following section, these estimates are consistent with the available public deficit data between 1977 and 2016.

42 To asses the effect on inflation given a 1% β_t increase, I considered inflation's impulse response function for each of the estimated $\bar{d}$. The effect of expectations on inflation that I present is the average of the impulse-response results across $\bar{d}$.

43 The estimation of $\nu = 0.1147$ implies that the weight agents give to their past expectations is 0.8853. Hence, if inflation is stable for only a few months β will not be controlled because past beliefs have more weight on expectations. Only if the inflation rate is stable for several consecutive months, then β will also become stable.

6 The Conquest of Mexican Inflation

This section presents the paper's main results: first, I present the predictions that the model has for fiscal deficits relative to output, expectations, and inflation between January 1969 and December 2016. Then, as a validation method, I contrast these predictions with data available on these variables and with the economic history of inflation and fiscal deficits discussed in Section 3.

6.1 Computing Fiscal Deficits, Expectations, and Inflation

Once the parameters are estimated ($\hat{\phi}$), fiscal deficit relative to output may be computed at each t using the assumption made for $d_t|\bar{d}_t, v_t$ and considering that $\{\bar{d}_t, v_t\}$ follow a discrete Markov process. However, since I have data on inflation, it would be more accurate to incorporate this information into the fiscal deficits estimation. Thus, instead of computing the unconditional fiscal deficit density $p(d_t|\hat{\phi})$, I estimate the conditional density of d_t given the INPC inflation sequence π^T and the estimated parameters $\hat{\phi}$: $p(d_t|\pi^T, \hat{\phi})$.

To approximate $p(d_t|\pi^T, \hat{\phi})$ I follow the heuristic algorithm described in Appendix C. Then, I consider the median of each density in order to construct the estimated fiscal deficit sequence $(\hat{d}_t)_{t=0}^{T}$ according to the model. Given this estimated fiscal deficit history, using (4), (8), and conditioning on earlier inflation rates, I can make inference about the joint distribution $p(\pi_t, \beta_t|\pi^T, \hat{\phi})$ in order to construct a sequence for inflation and expectations $(\hat{\beta}_t, \hat{\pi}_t)_{t=1}^{T}$ according to the model. I interpret $(\hat{d}_t, \hat{\beta}_t, \hat{\pi}_t)_{t=1}^{T}$ as the estimated historical relationship between these variables, which I discuss in the following section.

6.2 The Conquest of Mexican Inflation

Figure IV presents the main results of Sargent et al. (2009)'s model for Mexico's case, that is, the dynamics between: (i) fiscal deficits; (ii) inflationary expectations; (iii) observed inflation; and (iv) the probability of a regime change in $\bar{d}$. According to the model:

- During 1969 and 1972, marked as region **(1)** in Figure IV, inflation was low since the government had a fiscal deficit associated with the lowest median hidden state ($\bar{d}_3$). This is consistent with Mexico's economic history: during the 60s, fiscal deficit was controlled and inflation achieved the minimum rate that it had during the second half of the twentieth century (2.8% according to the model).

- Between 1973 and 1982, region **(2)** of the figure, the model predicts that fiscal deficit relative to output went from a low to a moderate median level. This prediction reflects the deficits that the Mexican government produced during those years as a consequence of the interventionist regulations it implemented to maintain the economic boost of the 60s. This hidden state change induced an inflation spike during 1972. Since the moderate median deficit remained constant for several years, inflation expectations de-anchored. Naturally, the observed inflation rate also presented an increase between 1973-1982. As shown in panel **(d)**, during 1976 the fiscal deficit had a slight probability to change to a higher median regime. As Section 3 explains, this would have occurred but the Cantarell oil discovery in 1976 allowed the Mexican government to remain in a moderate deficit state.

- During 1983, the model predicts an inflation rate that went over 80% as a result of a fiscal deficit rise: in this year it achieved the highest median level allowed by the model (9.8% of the GDP). As a consequence, the public's expected inflation rate presented an increase during 1983-1984.

- Region (3) of Figure IV presents evidence of a **cosmetic reform** in order to control inflation: during 1984 the government was able to arrest inflation from 85% to 56% (according to the model) due to a temporal reduction of its fiscal deficit. However, as shown by panel **(a)** and **(d)**, the median fiscal deficit between 1985-1987 remained at its highest value. This provoked an inflation spike during 1985 and, consequently, the public's expected inflation increased as well. This prediction of the model is consistent with Cárdenas (2015): the crisis presented during 1987 was a direct consequence of the government's unwillingness to reduce its deficit during 1982-1987.

- To arrest inflation, during 1988-1993, the government had to conduct a **structural** reform: between 1988 and 1993 (region **(4)** of the figure), fiscal deficit went from the highest possible median ($\bar{d}_1$), to a moderate level in 1989 ($\bar{d}_2$) and then in 1993 to a lower median ($\bar{d}_3$). This fiscal deficit reduction had an immediate impact on inflation and expectations. The model is also consistent with this episode of Mexico's economic history: in 1988 president de la Madrid made an agreement with several sectors of the economy in order to reduce fiscal deficit dramatically. This agreement was called Economic Solidarity Plan (*Pacto de Solidaridad Económica*). A similar fiscal consolidation plan was implemented during president Salinas de Gortari's term between 1988 and 1994.[44]

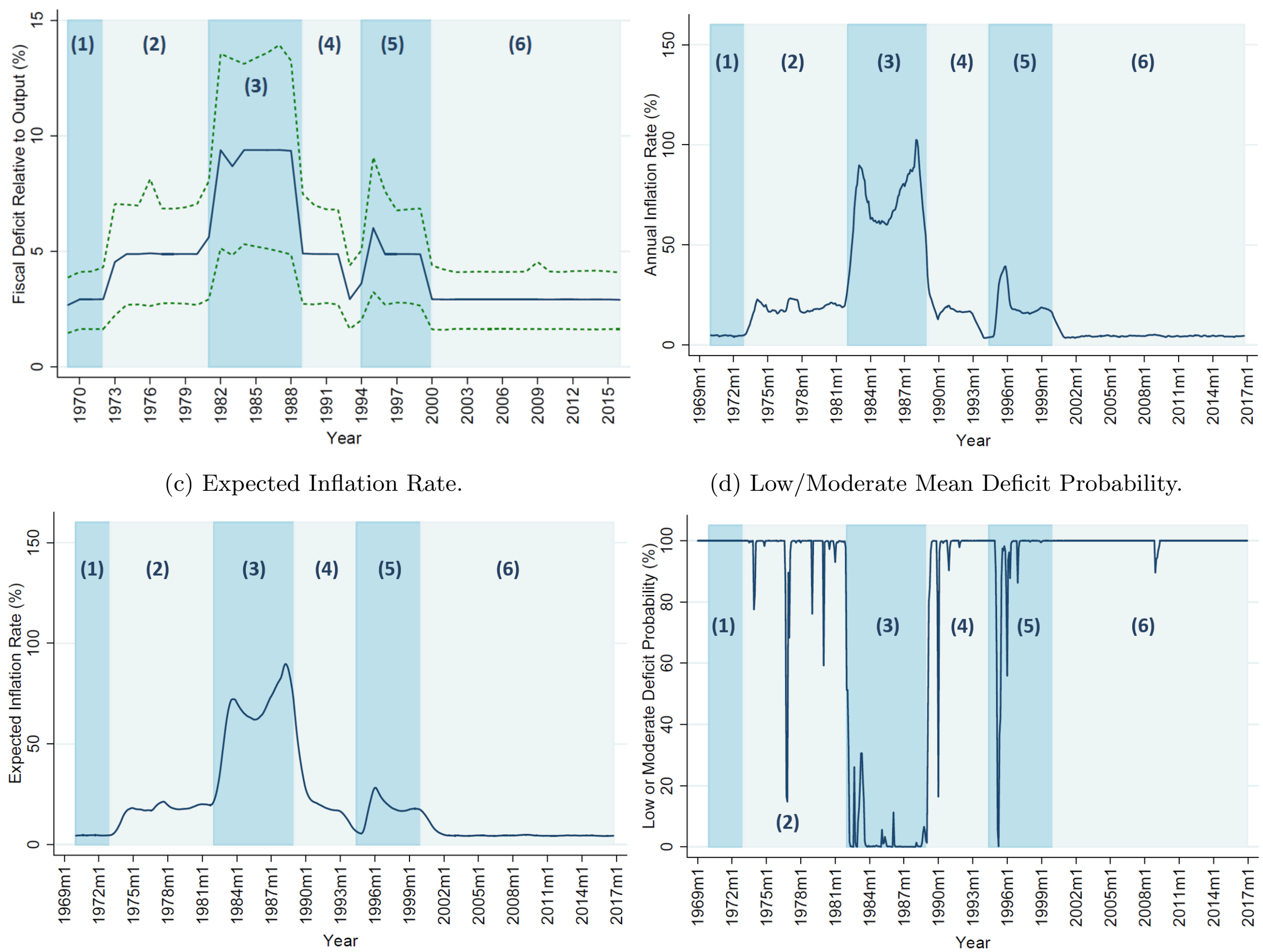
- The model suggests, in region **(5)**, that inflation presented a spike due to a fiscal deficit rise during 1994-1995. Actually, this elevated inflation was caused by a nominal exchange rate depreciation at the end of 1994 and the collapse of the financial sector in 1995.[45] Since the government received aid from several international institutions to cover all the debt it had in *Tesobonos*, the government's fiscal deficit did not explode. Hence, there is a discrepancy between the model's prediction about fiscal deficit during 1994-1995 and the data. According to the model, fiscal deficit relative to output was controlled by 1999.

- During the twentieth first century (region **(6)** of Figure IV) since inflation has been low and stable, the model predicts that fiscal deficit must be at its lowest median and variance hidden states ($\bar{d}_3$, v_2 respectively). The model also shows that the expected inflation rate has anchored to *Banco de México*'s inflation target: an inflation of 3% that can vary between 2% and 4%. Consequently, the model suggests that a necessary condition to anchor inflation and its expectations near the Central Bank's target is a permanently low fiscal deficit. The only year in which fiscal deficit had a slight probability of being at a higher median state was in 2009, year of the global financial crisis. However, since inflation after 2009 has been controlled, the model predicts that Mexico has remained in a low fiscal deficit regime.

44 For more details regarding these consolidation plans, see Cárdenas (2015).

45 Calvo and Mendoza (1996), Cole and Kehoe (1996), among others, analyze the causes and consequences of the 1995 crisis.

Figure IV: The Conquest of Mexican Inflation.



(a) Fiscal Deficit Relative to Output. (b) Inflation Rate. (c) Expected Inflation Rate. (d) Low/Moderate Mean Deficit Probability.

Notes: panel **(a)** plots the median fiscal deficit relative to output among the 10^{th} and 90^{th} percentile of the annual deficit distribution. Panel **(b)** shows the annual inflation rate predicted by the model. Panel **(c)** shows the public's expected inflation rate according to the CGE algorithm (4). Panel **(d)** plots $P[\bar{d}_t = \bar{d}_2 | \pi^T, \hat{\phi}] + P[\bar{d}_t = \bar{d}_3 | \pi^T, \hat{\phi}]$, where $\bar{d}_2$, and $\bar{d}_3$ represent the moderate and low median fiscal deficit hidden states.

Considering these sequences, Table III presents the average annual inflation implied by each hidden Markov state $\bar{d}$. When fiscal deficit has been low (according to the model this happened between 1969-1973 and 2000-2016) annual inflation has been 3.54% on average. A moderate fiscal deficit (presented during 1973-1981 and 1990-1999) implied an average annual inflation of 17.53%. Finally, a high fiscal deficit has been associated with an annual inflation of 79.41% (on average). The values presented on Table III may be interpreted as the long-run (or non-stochastic) inflation equilibria induced by each $\bar{d}$.

Table III: Median Fiscal Deficit and Long-Run Inflation.

Hidden Markov State	Median Deficit Relative to Output (%)	Average Annual Inflation (%)
$\bar{d}_3$	2.78	3.54
$\bar{d}_2$	4.76	17.53
$\bar{d}_1$	9.12	79.41

6.3 Model Predictions and Data

Having a sequence of actual inflation between January 1969 and December 2016 (π^T) and assuming a hidden Markov process for the evolution of fiscal deficits allows the model to make inference about the joint distribution $p(d_t, \pi_t, \beta_t|\pi^T, \hat{\phi})$ at each t to produce a sequence $(\hat{d}_t, \hat{\pi}_t, \hat{\beta}_t)_{t=1}^T$ that is consistent with the inflation data (conditional on the model's structure). As a way to validate the relationship implied by the model between $\{\hat{d}, \hat{\pi}, \hat{\beta}\}$, in this section I contrast each of these sequences with the available data. Additionally, I contrast the model's predictions about the monetary base growth rate with the Central Bank's historical data on this variable.

6.3.1 Inflation

Panel **(a)** of Figure V plots, between 1969 and 2016, the annual inflation rate according to the model and the historical INPC data. As shown in the panel, the model has a good approximation to the inflation data: the mean relative error between the model's prediction and the data is 19.4%.[46] Additionally, the model's inflation variance is 61.5% of the data's variance, a result consistent with Rogers and Wang (1994). In this paper the authors show, using a structural VAR, that approximately 60% of the inflation variance presented during 1977-1990 is explained by fiscal and monetary shocks.

6.3.2 Expected Inflation

Since 1999, Mexico's Central Bank (*Banco de México*) measures agents' expectations on several macroeconomic variables through a monthly survey made to approximately 40 specialists. Regarding inflation expectations, *Banco de México* asks to each specialist the inflation rate that they expect in the month of the survey, the following month, the following twelve months, and in the long-run. To be consistent with the model, I contrast the expected inflation rate generated by the model with the specialists' inflationary expectations for the following month after they are surveyed.[47]

46 The relative error between a model's prediction $\hat{y}_t$ and the data y_t is computed as $e_t = 100(\hat{y}_t - y_t)/y_t$. The mean relative error is the average, across t, of $|e_t|$.

47 In the model, the gross expected inflation rate at month t is defined as $\beta_t = P^e_{t+1}/P_t$. Hence, β_t represents the monthly inflation that agents expect between month t and month $t+1$.

In panel **(b)** of Figure V, I present the annualized expected inflation rate according to the model and the data. As this figure shows, between 2001-2016, the model's expected inflation sequence is part of the inter-quartile range of *Banco de México*'s survey. The mean relative error between the model's inflationary expectations and the data's median expected inflation rate is 14.07%. The model generates a sequence of expectations that accounts for 66.51% of the variance presented in the specialists' median expected inflation between 2001-2016.

As shown in the model and in the data, since 2001, the expected inflation rate is anchored near *Banco de México*'s inflation target: an inflation of 3% that may vary between 2% and 4%. According to the model, this behaviour of the expected inflation rate is a result of a low fiscal deficit after 2001.

6.3.3 Fiscal Deficit

Regarding fiscal deficit, the Mexican authorities compute two official measures: *Balance Público Tradicional* (BPT) and *Requerimientos Financieros del Sector Público* (RFSP). The BPT represents the difference between current and capital expenditures with total revenue of almost all the public sector. However, it does not consider the revenue/expenditures of *Banco de México* neither the public financial sector.[48] It is worth mentioning that, before 1990, the BPT is an historical construction made with the available data.

Since 1990 the Mexican Ministry of Finance, *Secretaría de Hacienda y Crédito Público* (SHCP), computes the RFSP which consider all the financial requirements that the government uses for its public policy at a federal level. This is a broader measure of fiscal deficit since it includes the BPT but also all the revenue/expenditures of the public financial sector that provide funds for public policy.[49] To contrast the estimated fiscal deficit with the data the RFSP are ideal since they represent a deficit that is more consistent with this variable's definition within the model. However, before 1990 the only available deficit measure is the BPT.

The BPT and RFSP are similar between 2003-2016 but show considerable differences during the 90s. Considering the events of the 1995 crisis, the Mexican government received aid from different international institutions to reduce its deficit. Nevertheless, the government had to aid the private financial sector of the economy that had declared in bankruptcy. This aid given to the private sector is not considered in the BPT but it is in in the RFSP. This partially explains the difference between the BPT and the RFSP during the 90s.

Panel **(c)** of Figure V displays the model's estimated fiscal deficit sequence, the BPT and the RFSP relative to the GDP during 1977 and 2016. As this figure shows, the model's estimated deficit has an acceptable fit to the BPT data before 1991 and to the RFSP after 1993. According to the data, the government produced fiscal surplus during 1991 and 1992. The model cannot match this feature of the data: since d_t has a log-normal distribution, the model's deficit cannot be negative. Additionally, the model also predicts a higher deficit during 1994-1996 compared with the data. Particularly, in 1995 the model estimates that fiscal deficit relative to output was 6.1% of GDP but the RFSP data shows that fiscal deficit relative to GDP was 2.5%. This difference occurs since I only consider inflation data to estimate fiscal deficits. Given that inflation during 1994-1996 was considerably high, the model can only explain this inflation spike as a consequence of a fiscal deficit

48 SHCP (2013) includes in the financial sector of the government all the trust funds, banks, among others that are administered by the Federal Government.

49 For example, during 1990-1998 the government had a trust fund called FOBAPROA whose objective was to insure private banks against overdue accounts in case of a financial crisis. If this fund gave some of its earning's to a private bank to cover its overdue accounts, this was not measured in the BPT but it was considered in the RFSP. All the methodology that the SHCP follows to compute the RFSP is detailed in SHCP (2015).

increase.

Between 1977 and 2016 the model's median deficit accounts for 53.7% of the variance presented in the fiscal deficit data, meanwhile, the relative mean prediction error is 39.74%.[50] These results are a consequence of the restrictions imposed to $\{\bar{d}, v\}$. If the model's deficit had more hidden Markov states to define it ($D > 3$ or $V > 2$) its fit to the fiscal deficit data could improve. However, this would considerably increase the estimation's computational cost.

6.3.4 Monetary Base Growth Rate

Given that the model's inflation rate is derived from a money demand function, the model generates a prediction about the monetary base growth rate between 1969 and 2016. Therefore, I can contrast this prediction with *Banco de México*'s data on this variable. In the model, the real balances demand equation ($M_t/P_t = 1 - \lambda\beta_t$) implies that the annual growth rate of the monetary base relative to output M_t is:

$$\frac{M_t}{M_{t-12}} = \frac{P_t}{P_{t-12}} \left(\frac{1 - \lambda\beta_t}{1 - \lambda\beta_{t-12}} \right). \tag{10}$$

As explained in Section 4, since θ can be interpreted as the inverse of the output growth factor, then:[51]

$$\frac{\hat{M}_t}{\hat{M}_{t-12}} = \frac{Y_t}{Y_{t-12}} \frac{P_t}{P_{t-12}} \left(\frac{1 - \lambda\beta_t}{1 - \lambda\beta_{t-12}} \right) = \frac{\pi_t^A}{\theta^{12}} \left(\frac{1 - \lambda\beta_t}{1 - \lambda\beta_{t-12}} \right), \tag{11}$$

where $\hat{M}_t$ represents the nominal balances at t and π_t^A is the annual gross inflation rate between $t-12$ and t. Panel **(d)** of Figure V displays the annual growth rate of the monetary base according to the model (i.e., using equation Equation (11)) and the data. I contrast the model's prediction about $\hat{M}_t/\hat{M}_{t-12}$ with that data and do not consider M_t/M_{t-12} since I do not have enough data on Mexico's GDP between 1969 and 2016. On the other hand, I do have data on Mexico's monetary base according to *Banco de México* since 1960.

The figure shows that the model approximates reasonably well the data's sequence, although, there are considerable differences between 1982-1985 and 1990-1992. The model's monetary base growth rate accounts for 82% of the variance presented in the data and the mean relative deviation between the model and the data is 22.1%. It is worth mentioning that *Banco de México*'s methodology regarding this variable had, between 1969 and 2016, two important changes: the first one in 1985 and, the second, in 1991. This could explain the difference between the model's prediction and the data (especially the 1991 difference).

50 For these results, I considered the BPT before 1991 and the RFSP after this year.

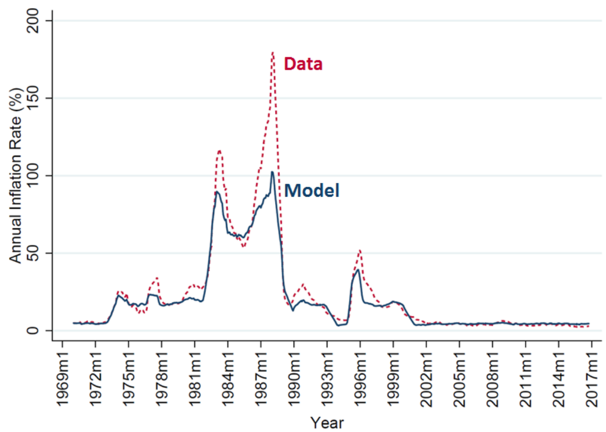
51 The parameter θ represents the persistence of the monetary base relative to output. However, it can also be interpreted as the inverse of the output growth factor. Let $M_t = \hat{M}_t/Y_t$ where $\hat{M}_t$ are the nominal balances at time t and Y_t is output. If D_t represents the fiscal deficit at time t then, the government budget constraint is: $\hat{M}_t = \hat{M}_{t-1} + P_t D_t$. Dividing this equation by Y_t then:

$$M_t = \frac{Y_{t-1}}{Y_t} M_{t-1} + P_t d_t = \theta M_{t-1} + P_t d_t.$$

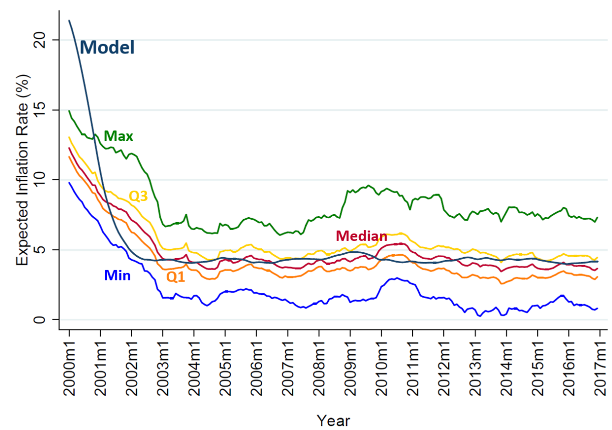
Therefore, θ can be interpreted as the inverse of the output growth factor.

Figure V: Model Predictions and the Available Data.

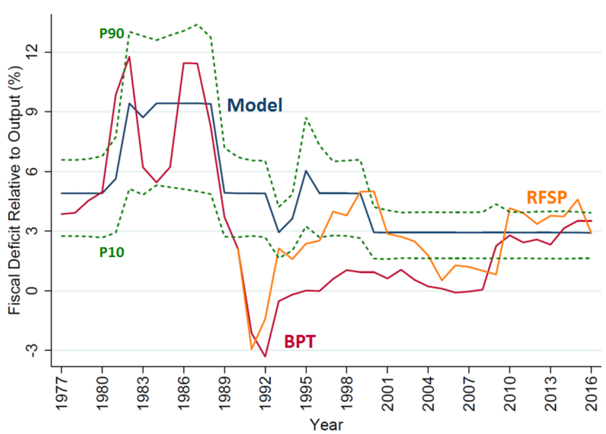
(a) Annual Inflation 1969 - 2016.



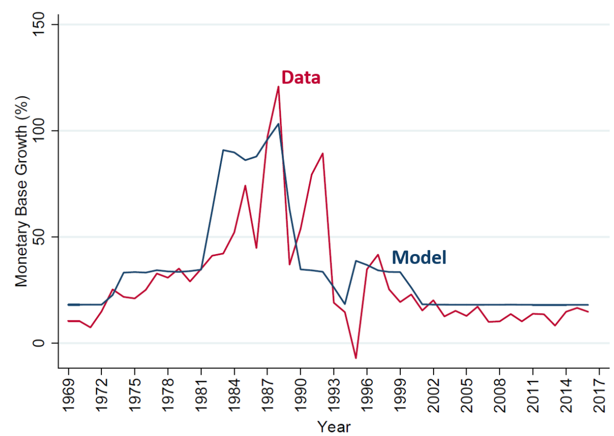
(b) Expected Inflation 2000 - 2016.



(c) Fiscal Deficit 1977 - 2016.



(d) Monetary Base 1969 - 2016.



Source: Banco de México, INEGI, and SHCP.

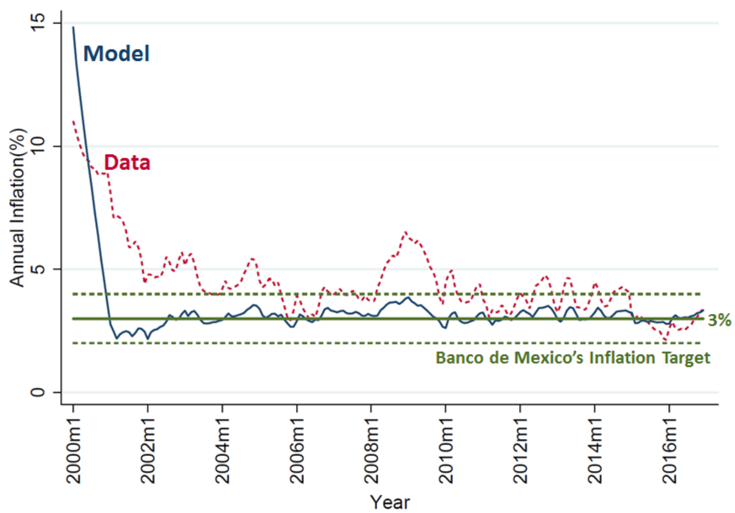
Notes: the series presented are - panel **(a)**: in blue/red the model/INPC annual inflation. Panel **(b)**: in navy blue, the model's expected annual inflation rate. In blue/ orange/ red/ yellow/ green the minimum/ first quartile/ median/ third quartile/ maximum of *Banco de México*'s expected inflation survey. Panel **(c)**: in blue the estimated fiscal deficit among the 10^{th}/ 90^{th} percentiles of the deficit distribution. In red/orange the BPT/RFSP relative to GDP. Panel **(d)**: In blue/red the model/data monetary base annual growth rate.

6.4 Mexico's Central Bank Autonomy

As Section 3 explains, a constitutional reform in 1993 made *Banco de México* an autonomous Central Bank. As a consequence, fiscal deficits cannot be financed by the Central Bank (thus, they are converted into public debt). Then, in 2002, *Banco de México* established an inflation-targeting policy that has as an objective an annual inflation rate of 3% that can float between 2% and 4%.

The model interprets these events as a permanent change in the deficit's median hidden state to $\bar{d}_3$ after 1999, allowing inflation to converge to *Banco de México*'s target as shown in Figure VI. However, the model predicts that inflation was within the Central Bank's target by 2001 while the data suggests that this occurred until 2006. As it is presented in this figure and in Figure IV, since the median hidden state is constant between 2000-2016 ($\bar{d}_3$) and inflationary expectations are anchored, the model's inflation rate has a limited variance. This suggests that, during these years, the model generates an inflation sequence that follows a stationary process, result that Chiquiar et al. (2007) proved for the actual inflation rate after 2002.

Figure VI: Annual Inflation According to the Model and Data.



Source: Banco de México and INEGI.

This figure suggests that, even though the model assumes fiscal dominance between 1969-2016, it reproduces the inflation rate behaviour between 2001-2016 by forcing fiscal deficits relative to output to be low and constant. Also, this prediction for fiscal deficits is consistent with the data as shown in panel **(c)** of Figure V.

To asses if the Central Bank autonomy had a structural effect on the model, I re-estimated it allowing the parameters to variate between 1969-1994 and 1994-2016. Then, using a Student's t-test, I conclude that only $\{\lambda, \nu\}$ are statistically different between 1969-1994 and 1994-2016 as it is shown in Table IV. The difference between $\{\lambda, \nu\}$ in each sample suggests that, after 1994, agents give a higher weight to observed inflation on their beliefs about future inflation (as suggested by the estimated ν). Consequently, inflation expectations are easily anchored after 1994 (relative to the period 1969-1994). Also, the post-1994 estimated λ implies that the expected inflation rate has lower impact on the price level, leading to an inflation with a reduced variance.

The model estimates a higher ν and a reduction in λ starting in 1994 since the inflation data used to estimate these parameters show a structural change, starting around 2001 as suggested by Chiquiar et al. (2007), and has become low, stable, and near 3%. The estimated change of $\{\lambda, \nu\}$ after 1994 is consistent, within the model, with this structural change presented in the data's inflation sequence.

Table IV: Parameter Variation Between 1969-1994 and 1994-2016.

Parameter	1969 - 1994	1994 - 2016	Statistically Different?
λ	0.7325	0.6984	Yes
ν	0.1187	0.1838	Yes
$\bar{d}_1$	0.0078	0.0070	No
$\bar{d}_2$	0.0043	0.0047	No
$\bar{d}_3$	0.0029	0.0021	No
v_1	0.1013	0.1340	No
v_2	0.0220	0.0207	No

Notes: the parameters of each sample were estimated using the algorithm described in Section 5.1. To test if the estimated parameters are statistically different between the samples, I used a Student's t-test. The parameters $\{\lambda, \nu\}$ are statistically different at a 1% confidence level.

Even though some estimated parameters are statistically different when considering two samples, the sequences $(\hat{d}_t, \hat{\pi}_t, \hat{\beta}_t)_{t=1}^T$ are negligibly different if one considers the parameters shown in Table II or in Table IV. Formally, using the test proposed in Diebold and Mariano (2002), I conclude that each inflation sequence $(\hat{\pi}_t)_{t=1}^T$ generated by the baseline/two-sample model are, statistically, the same forecast of Mexico's inflation rate between 2002-2016.[52]

This exercise suggests that, even though an important structural change occurred in 1994, the baseline model presented in this paper is able to account for Mexico's inflation rate after 1994 and its prediction for fiscal deficits (that they are currently in a low regime and have a near-zero probability to change to a higher state) is relatively similar to what actually occurred (*Banco de México*'s autonomy).

52 The hypothesis test proposed in Diebold and Mariano (2002) allows to assess if two forecasts $(y_{it}, y_{jt})_{t=1}^T$ about a series $(y_t)_{t=1}^T$ are statistically different. Defining $e_{kt} = y_{kt} - y_t$ for $k \in \{i, j\}$ and considering a loss-function $g(e)$, the null hypothesis in the Diebold-Mariano test is $H_0 : \mathbb{E}[g(e_{it}) - g(e_{jt})] = 0$. These authors construct a test statistic that involves the autocorrelations of the forecasts and show that, if the time series considered are covariance stationary and short memory, it has a Student's t distribution. Then, they construct a test statistic that, under the same assumptions, is asymptotically $N(0, 1)$. I use this test with each sequence $(\pi_t)_{t=1}^T$ because: (i) Chiquiar et al. (2007) prove that, since 2002, Mexico's inflation rate follows a stationary process; and (ii) the variance of the inflation sequence induced by each model is lower than the data's variance.

7 Inflation Predictions for 2017-2021

In this section I present an extension of the Sargent et al. (2009) framework: given the model's structure and the parameter estimation, I construct predictions for the inflation rate of 2017-2021 considering several future paths for fiscal deficits relative to output. Additionally, I contrast each fiscal deficit forecast with the official projection made for the RFSP during 2017-2021: Mexico's Fiscal Authority estimates in SHCP (2016) that their deficits will evolve from 2.9% in 2016 to 2% by 2021.

Since inflation has been low during 2016 (and actually achieved an historical minimum during this year), the model predicts that the deficit's median hidden state is low ($\bar{d}_3$) and that it has a low variance parameter (v_2). Therefore, I present predictions for the inflation rate considering three alternative paths for the evolution of fiscal deficits:

1. By December 2021, fiscal deficit will have a low median ($\bar{d}_3$) but a higher variance (v_1).
2. By December 2021, fiscal deficit will have a moderate median ($\bar{d}_2$) and a low variance (v_2).
3. By December 2021, fiscal deficit will have a moderate median ($\bar{d}_2$) and a high variance (v_1).

Figure VII shows, for each of the three potential fiscal deficit trajectories, a fan chart for the future deficit levels and its induced inflation rate. Appendix D explains the complete procedure I follow to construct these predictions.

If by December 2021 fiscal deficit relative to output has a low median and a higher variance parameter, then:

- During 2017 fiscal deficit relative to output will be between 1.83% and 3.98% of GDP implying an annual inflation rate between 2.88% and 7.74%. However, as the fan chart suggests, inflation is likely to be around 4.71%.
- By 2021, this fiscal deficit evolution implies an inflation rate between 3.21% and 6.38%. The inflation distribution of 2021 has a median of approximately 3.87%.
- This fiscal deficit evolution includes the projection made by the SHCP for the RFSP during 2017-2021. Also, this deficit path guarantees (according to the model) that in the long-run inflation and its expectations will be within *Banco de México*'s inflation target.

If by December 2021 fiscal deficit relative to output has a moderate median and a low variance parameter, then:

- During 2017 fiscal deficit relative to output will be between 2.18% and 5.52%. These deficit levels imply an annual inflation rate between 3.12% and 15.80% with a median of 5.14%.
- This deficit evolution implies an annual inflation for 2021 between 14.03% and 18.85%. This inflation distribution has as a median 17.87%.
- This path for fiscal deficit does not include the projection that the SHCP has made for the RFSP. It is worth mentioning that, if the RFSP follows a trajectory as the one showed in panel **(b)** then the government's debt will be over 60% of the GDP by 2021.

If by December 2021 fiscal deficit relative to output has a moderate median and a high variance parameter, then:

- The model predicts a fiscal deficit at the end of 2017 between 2.05% and 5.58%. This fiscal deficit implies an annual inflation rate between 3.08% and 15.95%. However, the inflation distribution has a median of 5.21%.

- For 2021, the annual inflation rate will be between 11.95% and 21.39% having a median of 17.76%.

Panels **(b)**, **(c)**, **(e)**, and **(f)** show the sort and long-run effects that high fiscal deficits have on inflation according to the model. According to these figures, a change in the median hidden state has a negligible short-run impact on inflation since the expected inflation rate is slightly altered.[53] However, if fiscal deficit continues in a moderate or high level for a prolonged period, this de-anchors expectations and provokes an inflation spike.

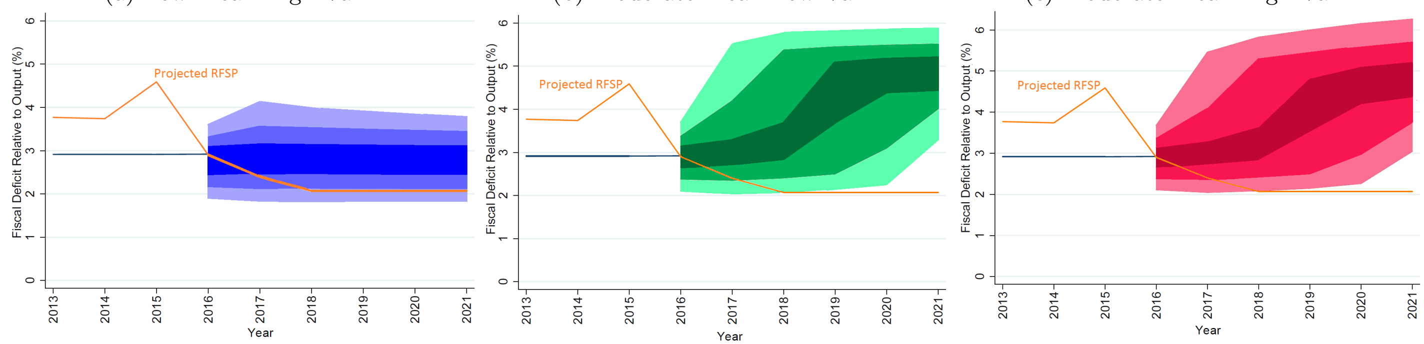
The predictions made by the model (especially the predictions that are induced by a moderate median deficit) could be seen as catastrophic considering that the annual inflation of 2016 was 3.36%. However, the model associates a moderate deficit to the inflation rates presented during 1975-1982 and 1994-1999, in whose inflation was 17.53% on average. Additionally, although the model does not explicitly consider that since 1994 Mexico has an autonomous Central Bank (hence, nowadays it is the Fiscal Authority that has to adjust its deficit to prevent a debt crisis) Kocherlakota (2012), among other authors, suggest that the Central Bank independence could be questioned by the economy's agents if the government generates a considerable deficit that translates into an increasing debt. Under this scenario, agents could anticipate a possibility that the Central Bank aids the Fiscal Authority to pay its debt in the future and increase their current inflation expectations, inducing an inflation spike. Hence, even though Mexico does not have a fiscal dominance regime any more, the fiscal policy can still influence the price level and produce a de-anchor of the expected inflation rate. This mechanism could generate an effect of fiscal policy on inflation during 2017-2021 as the one predicted by the model, especially if fiscal deficit has an increasing trajectory.

53 The estimated ν suggests that inflation must be high for several consecutive periods to substantially increase β.

Figure VII: Predictions for 2017-2021.

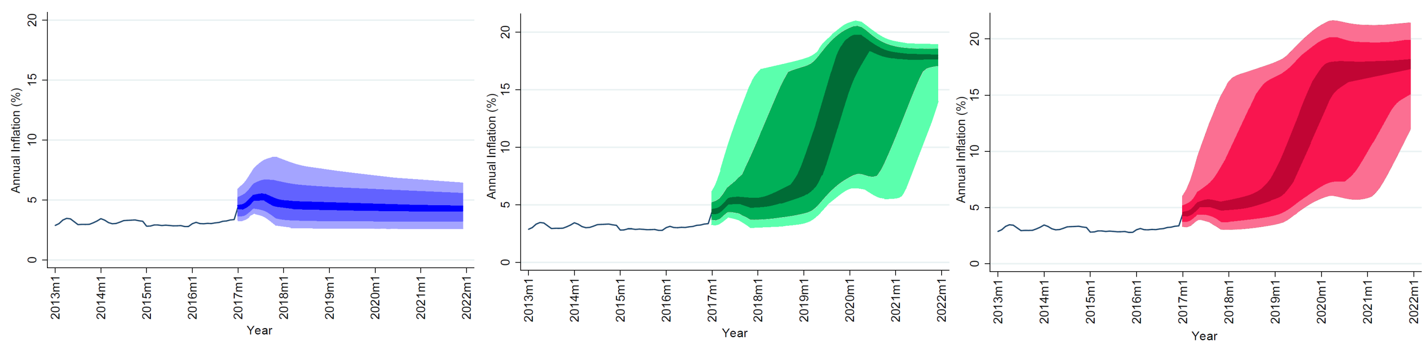
Fiscal Deficit Considering an Evolution Towards a:

(a) Low Mean-High Var. (b) Moderate Mean-Low Var. (c) Moderate Mean-High Var.



Inflation Rate with an Evolution of Fiscal Deficit Towards a:

(d) Low Mean-High Var. (e) Moderate Mean-Low Var. (f) Moderate Mean-High Var.



Source: SHCP (2016) for the 2017-2021 RFSP projection.

Notes: panels **(a),(b),(c)** plot the distribution of annual deficit during 2017 and 2021 assuming that in December 2021 with probability 1 the state of nature is the one indicated on the panel's title. The orange line on these panels represents the projection that the SHCP has on the RFSP for the following 5 years. Panels **(d),(e),(f)** plot the inflation rate fan chart considering each percentile of the fiscal deficit distribution shown above each panel and the expected inflation rate that each percentile induces.

8 Conclusions

I present a hidden Markov model that explains the evolution of Mexico's inflation rate between 1969 and 2016 as a result of an estimated sequence of inflationary expectations and fiscal deficits relative to GDP. According to the estimation results, when fiscal deficit relative to output has been around 2.78%, annual inflation has been on average 3.54%. Whenever fiscal deficit was at a moderate level (4.76% of GDP) it has induced an inflation around 17.53%. Meanwhile, a high fiscal deficit (9.12% of GDP) has provoked the highest inflation episodes that Mexico had during the past 48 years, with an average of 79.41%. This relationship estimated by the model between inflation and fiscal deficits is consistent with the data as shown in Figure V.

A higher fiscal deficit increases inflation, although, the short-run effect is limited. Only a prolonged deficit increase has considerable consequences on inflation since this de-anchors inflationary expectations. Given that in Mexico the expected inflation rate has an important impact on the price level (as suggested by the estimated λ), a destabilization of this variable induces an inflation spike. On average, a 1 percentage point increase in expectations induces an inflation rise of 0.56 percentage points. Additionally, to anchor expectations inflation must be controlled for several consecutive periods (as suggested by the estimated ν). Therefore, the model suggests that an effective solution to control high inflation in Mexico's case has been to conduct a structural reform, which reduces fiscal deficit permanently. A cosmetic reform, as the one implemented in 1984, has not been effective since a temporary reduction of deficits and inflation is not enough to anchor expectations. The model implies that Mexico had a structural reform during 1988, after which the fiscal deficit went from having a high median to the lowest value allowed by the model.

Considering the estimated historical relationship between inflation, expectations, and deficits, I present forecasts for the 2017-2021 inflation rate. As suggested by my results, if the Mexican government has a growing fiscal deficit during the following years, even though in the short-run (2017) inflation may be moderate, the model estimates that inflation could become considerably high by 2021: in the worst case scenario, inflation could be between 11.95% and 21.39% with a median of 17.76%. Even though nowadays Mexico does not have a fiscal dominance regime as the one assumed in the model, these predictions may reflect the potential inflation rates should the fiscal deficit grow between 2017-2021: as suggested by the vast literature about the interaction between fiscal and monetary policy, if agents observe a high fiscal deficit that translates into an increasing debt, they could increase their inflation expectations causing an inflation spiral.[54] Consequently, this model suggests that, in order to maintain a controlled inflation within the Central Bank's target, the Fiscal Authority should promote an effective management of the public finances that induces a moderate fiscal deficit.

54 The channels through which an increasing deficit or debt could translate into an inflation spike in a context of Central Bank independence is discussed in papers like Sargent and Wallace (1981), Cochrane (2001), Kocherlakota (2012), Sims (2016), and Bianchi and Ilut (2017).

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A Adaptive vs Rational Expectations

Sargent and Wallace (1973) say expectations about a variable are rational if "they depend, in the proper way, on the same things that economic theory says actually determine that variable". They criticise adaptive expectations because "usually, but not always, assume that the people whose expectations count are ignorant of the economic forces governing the variable they are trying to predict". Marcet and Nicolini (2003) also point out several critiques to adaptive expectations: (i) they involve too many degrees of freedom; (ii) they led to possible irrational expectations and; (iii) assume expectations that are exogenous to the model.[55] This appendix points out some of the implications that a rational expectations algorithm has in the model presented in Section 4.[56] Besides, I contrast the main differences induced in the dynamics of the model between this type of expectations and CGE.

One way to model that agents are rational when forming their beliefs on future inflation is to suppose:

$$\beta_{t+1} = \mathbb{E}_t[\pi_{t+1}|\bar{d}_t, v_t]. \tag{12}$$

If agents are rational, they condition their expectations on the median ($\bar{d}_t$) and variance (v_t) hidden states of fiscal deficit because the stochastic process that governs these parameters is known to agents when they are rational. This is an important difference with adaptive expectations. Assuming CGE does not require agents to condition their expectations on $\{\bar{d}_t,\ v_t\}$ because they always update their beliefs using only the information they have observed.[57]

Assuming rational expectations also affects the dynamics between the gross inflation rate of two consecutive periods (π_t, π_{t+1}) as a function of β_t. Panel **(a)** of Figure VIII plots the difference $\pi_{t+1} - \pi_t$ as a function of β_t assuming that β_{t+1} is determined according to (12). This figure considers the same median and variance state for the fiscal deficit in t and $t+1$. As this figure shows, there is only one value of β_t that induces a constant inflation (and expectations) over time: β_1. As the figure suggests, β_1 is a stable equilibrium. Thus, if fiscal deficit remains with the same median and variance level, $\pi_{t+1} - \pi_t$ will converge to zero and $\{\beta_t, \pi_t\}$ to β_1.

With rational expectations, even with a high expected inflation ($\beta_t > \beta_1$), agents will not allow their expectations to provoke an escape. Their expectations will always adjust and converge to β_1. However, the government could prevent expectations to converge to a high inflation equilibrium

55 Marcet and Nicolini (2003) cite two common critiques to adaptive expectations. The first one is "any economic model can match any observation by choosing expectations appropriately" and the second is "economic agents do not make systematic mistakes".

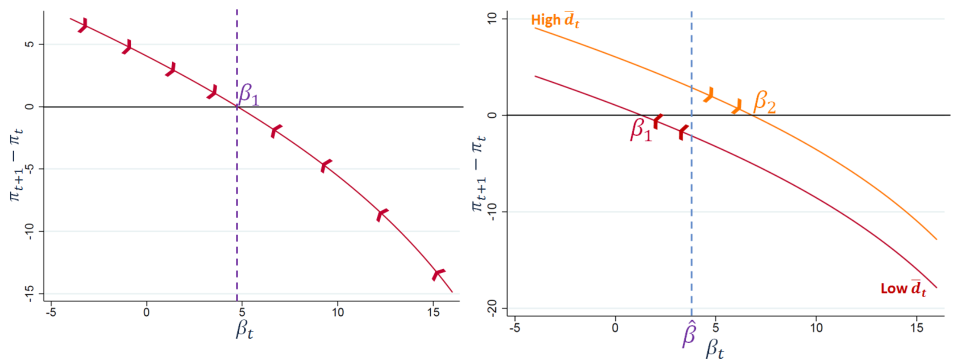
56 Barbosa (2016) makes a summary of different models of hyperinflation and the expectations each model assumes.

57 Branch (2004) argues rational expectations requires agents to acquire more information than other algorithms. Since information is costly, agents could rationally choose not to update their beliefs following a rational expectations algorithm.

by permanently reducing its fiscal deficits as shown in panel **(b)** of Figure VIII. This figure plots $\pi_{t+1} - \pi_t$ as a function of β_t for two different $\bar{d}$: low and high. Assuming $\beta = \hat{\beta}$ and that the median fiscal deficit is high, if the government continues with this deficit level then the model will induce a convergence towards a high inflation equilibrium (β_2). However, if the government reduces its deficit permanently, it will change the dynamics on inflation and its expectations inducing a convergence towards β_1.

Figure VIII: Dynamics Induced By Rational Expectations.

(a) Inflation and Expectations. (b) Fiscal Deficit and Expectations.



Notes: these figures consider $\beta_{t-1} = 1.02$ and the estimated parameters with Mexican data shown in Table II.

Figure VIII points out an important difference between rational expectations and CGE: when agents use the CGE algorithm, if the current inflation rate induces a high β_t then this could provoke an escape dynamics where the dynamics between inflation and its expectations are unbounded. With rational expectations, even with a considerably high fiscal deficit, agents always adapt their expectations to prevent an hyperinflation spiral. If fiscal deficit is high, rational expectations implies a stable equilibrium with a high inflation rate and no escapes. Empirically, the literature about rational expectations documents that hyperinflation episodes do not have an indefinite duration since high fiscal deficits financed throughout money creation cannot occur for a prolonged period. Hence, agents know that a high inflation episode will eventually end, therefore, it is not rational to expect a higher inflation rate in the long-run. Fischer et al. (2001) documents some stylized facts about high/hyperinflation episodes and their duration.

Even though it is standard to suppose rational expectations, recent literature tries to argue that agents do not necessarily follow this type of algorithms. For example, Malmendier and Nagel (2015) using micro data of an inflation expectations survey argue that people take more into account recent information to form their expectations when they are young. Older people tend to be more rational because since they understand more about the price formation process. In this sense, a CGE algorithm is ideal to approximate expectations about inflation if one interprets ν as the proportion of the economy that only considers past information to form their beliefs (the "young" ones), and $1 - \nu$ as the population's fraction that sticks to their beliefs (that could be formed rationally).

Branch (2004) develops a micro-founded model where agents rationally choose not to update their beliefs according to a rational expectations algorithm because the information it requires is too costly (rational expectations algorithms usually require a large amount of information). Barbosa et al. (2006) point out that "in any standard macro model of hyperinflation, it is necessary to impose a deviation from rational expectations and/or to violate the intertemporal government budget constraint for the model to generate explosive hyperinflation paths".

Finally, Sargent et al. (2009) argue that, in the context of hyperinflation models, "an adaptive expectations version of the model shares steady states with the rational expectations version, but has more plausible out-of-steady state dynamics". Besides, rational expectations may induce multiple equilibria that are hard to compute.[58] Given the computational problem rational expectations may induce and that some Latin American countries have experienced hyperinflation episodes with escapes, feature that a strictly rational expectations model cannot account for, Sargent et al. (2009) decided to use CGE in their model.

B The Likelihood Function

In this section I detail the procedure I follow to construct $p(\pi^T|\phi)$. For the remainder of this appendix let $p(\cdot)$ represent a density function for a certain random variable, $P[A]$ denotes the probability of the event A, and x^T represents a T period sequence for x: $x^T = (x_0, x_1, x_2, \ldots, x_T)$.

To construct $p(\pi^T|\phi)$, because of the Conditional Product Law, it can be decomposed as:[59]

$$p(\pi^T|\phi) = \prod_{t=1}^{T} p(\pi_t|\pi^{t-1}, \phi). \tag{13}$$

Since there are D possible states for $\bar{d}$ and V states for v, using the Law of Total Probability, $p(\pi_t|\pi^{t-1}, \phi)$ can be written as:[60]

$$p(\pi_t|\pi^{t-1}, \phi) = \sum_{i=1}^{D} \sum_{j=1}^{V} p(\pi_t|\pi^{t-1}, \bar{d}_i, v_j, \phi) P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^{t-1}, \phi]. \tag{14}$$

Each density $p(\pi_t|\pi^{t-1}, \bar{d}_i, v_j, \phi)$ can be computed using (8) and the assumption made on the distribution of d_t: when $\bar{d}_t, v_t$ are given, d_t has a log-normal distribution with mean $log(\bar{d}_t)$ and variance parameter v_t. In probability terms, $p(\pi_t|\pi^{t-1}, \bar{d}_i, v_j, \phi)$ can be seen as the density function associated to the transformation of d_t into π_t. Appendix B.1 explains how to compute this transformation and construct $p(\pi_t|\pi^{t-1}, \bar{d}_i, v_j, \phi)$. To compute $P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^{t-1}, \phi]$, I followed the algorithm suggested by Sims et al. (2006), that is a recursive method based on Bayes Theorem. This algorithm is detailed in Appendix B.2.

B.1 Conditional Inflation Density

In this section I explain how to compute, at each t and for every $\{\bar{d}, v\}$, the conditional probability density $p(\pi_t|\pi^{t-1}, \hat{\phi}, \bar{d}_t = \bar{d}, v_t = v)$ that is used to construct $p(\pi^T|\phi)$.

58 To compute expectations following a rational expectations algorithm, one has to use an iterative method whose convergence is not guaranteed. An iterative method is required because $\mathbb{E}_t[\pi_{t+1}|\bar{d}_{t+1}, v_{t+1}]$ is a function of β_{t+1} (since π_{t+1} depends on this variable).

59 The conditional product law states the following: if X, Y are two random variables then the joint density $p(X, Y)$ can be decomposed as $p(X, Y) = p(X|Y)p(Y)$ where $p(X|Y)$ is the conditional density function of X given Y. By induction, if $X_1, X_2, \ldots, X_m$ are random variables then the conditional product law states that: $p(X_1, X_2, \ldots, X_m) = \prod_{j=1}^{m} p(X_j|X^{j-1})$.

60 The law of total probability states: if A is an event and $B_1, B_2, \ldots, B_q$ is a partition of the sample space then: $P[A] = \sum_{i=1}^{q} P[A|B_i]P[B_i]$.

Let X be a random variable such that $\log(X) \sim N(\mu, \sigma^2)$. Then, the probability density function (PDF) of X is:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}x} e^{-\frac{(\log(x)-\mu)^2}{2\sigma^2}} \quad \text{if} \quad x > 0. \tag{15}$$

In the model considered in this paper, there are two variables that follow a log-normal distribution: d_t and $\pi_t^\star$.[61] The conditional PDF of each variable is:[62]

$$f_{d_t}(d_t|\bar{d}_t, v_t) = \frac{1}{v_t\sqrt{2\pi}d_t} e^{-\frac{(\log(d_t)-\log(\bar{d}))^2}{2v_t^2}} \quad \text{if} \quad d_t > 0, \tag{16}$$

$$f_{\pi_t^\star}(\pi_t^\star|\bar{\pi}_t, v_\pi) = \frac{1}{v_\pi\sqrt{2\pi}\pi_t^\star P[\pi_t^\star < \delta]} e^{-\frac{(\log(\pi_t^\star)-\log(\bar{\pi}_t))^2}{2v_\pi^2}} \quad \text{if} \quad 0 < \pi_t^\star < \delta. \tag{17}$$

Assuming $1-\lambda\beta_{t-1} > 0$ and $\delta(1-\lambda\beta_t - d_t) > \theta(1-\lambda\beta_{t-1})$, the inflation rate at each t is determined according to:

$$\pi_t = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t - d_t}. \tag{18}$$

Since d_t is a random variable and π_t is a function of d_t (as shown in (18)), using (16) and the inverse transformation method,[63] one can show that the PDF of π_t conditional on $1-\lambda\beta_{t-1} > 0$, $\delta(1-\lambda\beta_t - d_t) > \theta(1-\lambda\beta_{t-1})$, π^{t-1}, $\bar{d}_t$, and v_t is:[64]

$$f_{\pi_t}(\pi_t|\pi^{t-1}, \bar{d}_t, v_t) =$$

$$\frac{\theta(1-\lambda\beta_{t-1})}{v_t\sqrt{2\pi}[\pi_t(1-\lambda\beta_t) - \theta(1-\lambda\beta_{t-1})]\pi_t} e^{-\frac{(\log((1-\lambda\beta_t)\pi_t - \theta(1-\lambda\beta_{t-1})) - \log(\pi_t) - \log(d_t))^2}{2v_t^2}}. \tag{19}$$

Whenever $1-\lambda\beta_{t-1} \leq 0$ or $\delta(1-\lambda\beta_t - d_t) \leq \theta(1-\lambda\beta_{t-1})$, since inflation will be determined randomly, the conditional PDF of π_t given $\{\pi^{t-1}, \bar{d}_t, v_t\}$ is (17). Therefore, the PDF of π_t given $\pi^{t-1}, \bar{d}_t, v_t$ is:[65]

$$\begin{aligned} p(\pi_t|\pi^{t-1}, \phi, \bar{d}_t, v_t) = &\left(\mathcal{X}_{\{1-\lambda\beta_{t-1}\leq 0\}} + \mathcal{X}_{\{1-\lambda\beta_{t-1}>0\}} P\left[\pi_t = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t - d_t} > \delta\right]\right) f_{\pi_t^\star}(\pi_t^\star|\bar{\pi}_t, v_\pi) \ + \\ &\mathcal{X}_{\{1-\lambda\beta_{t-1}>0\}} \mathcal{X}_{\left\{\frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t} < \pi_t < \delta\right\}} f_{\pi_t}(\pi_t|\pi^{t-1}, \bar{d}_t, v_t) \ \text{ if } \ 0 < \pi_t < \delta. \end{aligned} \tag{20}$$

The following proposition will show that (20) is indeed a PDF.

Proposition B.1 *Consider $p(\pi_t|\pi^{t-1}, \phi, \bar{d}_t, v_t)$ as shown in (20), then:*

$$\int_0^\delta p(\pi_t|\pi^{t-1}, \phi, \bar{d}_t, v_t) d\pi_t = 1.$$

61 $\pi_t^\star$ is inflation according to the model when it is determined randomly, i.e whenever $1-\lambda\beta_{t-1} \leq 0$ or, $\delta(1-\lambda\beta_t - \gamma d_t) \leq \theta(1-\lambda\beta_{t-1})$.

62 It can be easily shown that $P[\pi_t^\star \leq \delta] = \Phi(\frac{\log(\delta)-\log(\bar{\pi}_t)}{v_\pi})$, where $\Phi(\bullet)$ is the cumulative distribution function of a standard normal variable.

63 The inverse transformation method (ITM) can be used when one knows the PDF of a random variable X and wants to compute the PDF of another variable Y such that $Y = g(X)$. The ITM states: if g is differentiable at all x in the support of X and has an inverse (g^{-1}) then: $f_Y(y) = \left|\partial g^{-1}/\partial y\right| f_X(g^{-1}(y))$. In this model, the function g that transforms d_t to π_t is (18) and its inverse is $d_t = \frac{(1-\lambda\beta_t)\pi_t - \theta(1-\lambda\beta_{t-1})}{\pi_t}$ which is clearly differentiable.

64 Notice that when $\pi^{t-1} = (\pi_0, \pi_1, \ldots, \pi_{t-1})$ is given, because of CGE, the sequence $\beta^t = (\beta_0, \beta_1, \ldots, \beta_t)$ is also given.

65 Using (16) it can be shown that $P[\pi_t > \delta] = P\left[d_t > \max\left\{1-\lambda\beta_t - \frac{\theta(1-\lambda\beta_{t-1})}{\delta}, 0\right\}\right]$.

Proof:

Consider the following cases: $1-\lambda\beta_{t-1} \leq 0$ or $1-\lambda\beta_{t-1} > 0$. If $1-\lambda\beta_{t-1} \leq 0$, since $f_{\pi_t^\star}(\pi_t^\star|\bar{\pi}_t, v_\pi)$ is a PDF with support $[0,\delta]$, then:

$$\int_0^\delta p(\pi_t|\pi^{t-1}, \phi, \bar{d}_t, v_t)d\pi_t = \int_0^\delta f_{\pi_t^\star}(\pi_t^\star|\bar{\pi}_t, v_\pi)d\pi_t^\star = 1.$$

In the case where $1-\lambda\beta_{t-1} > 0$, considering $\min_{\pi_t}$ as the minimum value for the model's inflation (when $d_t = 0$), and $\max_{d_t}$ as the maximum value that the deficit can be without inducing $\pi_t > \delta$, that is:

$$\min_{\pi_t} = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t},$$

$$\max_{d_t} = \max\left\{1-\lambda\beta_t - \frac{\theta(1-\lambda\beta_{t-1})}{\delta}, 0\right\},$$

it can be shown that:

$$\begin{aligned}
\int_0^\delta p(\pi_t|\pi^{t-1}, \phi, \bar{d}_t, v_t)d\pi_t &= P\left[\pi_t > \delta\right]\int_0^\delta f_{\pi_t^\star}(\pi_t^\star|\bar{\pi}_t, v_\pi)d\pi_t \\
&+ \int_0^\delta \mathcal{X}_{\left\{\min_{\pi_t} < \pi_t < \delta\right\}} f_{\pi_t}(\pi_t|\pi^{t-1}, \bar{d}_t, v_t)d\pi_t \\
&= P\left[\pi_t > \delta\right] + \int_{\min_{\pi_t}}^\delta f_{\pi_t}(\pi_t|\pi^{t-1}, \bar{d}_t, v_t)d\pi_t.
\end{aligned}$$

Since $\pi_t = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t - d_t} \in (\min_{\pi_t}, \delta]$ if and only if $d_t \in (0, \max_{d_t}]$, then:

$$\begin{aligned}
\int_0^\delta p(\pi_t|\pi^{t-1}, \phi, \bar{d}_t, v_t)d\pi_t &= P\left[\pi_t > \delta\right] + \int_{\min_{\pi_t}}^\delta f_{\pi_t}(\pi_t|\pi^{t-1}, \bar{d}_t, v_t)d\pi_t \\
&= P\left[d_t > \max_{d_t}\right] + \int_0^{\max_{d_t}} f_{d_t}(d_t|\bar{d}_t, v_t)dd_t \\
&= P\left[d_t > \max_{d_t}\right] + P\left[d_t \leq \max_{d_t}\right] \\
&= 1.
\end{aligned}$$

B.2 Sims et al. (2006) Algorithms

In this paper, I use two recursive algorithms proposed by Sims et al. (2006) that are useful to handle hidden Markov models. These algorithms allow me to compute the probabilities associated with the hidden part of the model $(\bar{d}, v)$ given partial or total information about the endogenous variables (π^t, π^T).

First Algorithm

The first algorithm I consider is useful to compute $P[\bar{d}_t = \bar{d}, v_t = v|\pi^{t-1}, \hat{\phi}]$, that is, the probability that in period t the median deficit state is $\bar{d}$ and the variance is v given the observed inflation rate up to period $t-1$ $(\pi_0, \pi_1, \ldots, \pi_{t-1})$ and the parameter estimation $(\hat{\phi})$.

The Law of Total Probability implies, for each $\bar{d}, v$:[66]

$$P[\bar{d}_t = \bar{d}, v_t = v|\pi^{t-1}, \hat{\phi}] = \sum_{\bar{d}_{t-1}} \sum_{v_{t-1}} P[\bar{d}_t = \bar{d}, v_t = v|d_{t-1}, v_{t-1}] P[\bar{d}_{t-1}, v_{t-1}|\pi^{t-1}, \hat{\phi}]. \tag{21}$$

Because the process for $\bar{d}$ is independent of the process that v follows, then:

$$P[\bar{d}_t = \bar{d}, v_t = v|\bar{d}_{t-1}, v_{t-1}] = P[\bar{d}_t = \bar{d}|\bar{d}_{t-1}] P[v_t = v|v_{t-1}]. \tag{22}$$

In this equation, $P[\bar{d}_t = \bar{d}|\bar{d}_{t-1}]$, $P[v_t = v|v_{t-1}]$ represent the transition probabilities that form Q_d, Q_v respectively. $P[\bar{d}_{t-1}, v_{t-1}|\pi^{t-1}, \hat{\phi}]$ is computed using Bayes Theorem:

$$P[\bar{d}_{t-1}, v_{t-1}|\pi^{t-1}, \hat{\phi}] = \frac{p(\pi_{t-1}|\pi^{t-2}, \bar{d}_{t-1}, v_{t-1}, \hat{\phi}) P[\bar{d}_{t-1}, v_{t-1}|\pi^{t-2}, \hat{\phi}]}{\sum_{\tilde{d}_{t-1}} \sum_{\tilde{v}_{t-1}} p(\pi_{t-1}|\pi^{t-2}, \tilde{d}_{t-1}, \tilde{v}_{t-1}, \hat{\phi}) P[\tilde{d}_{t-1}, \tilde{v}_{t-1}|\pi^{t-2}, \hat{\phi}]}, \tag{23}$$

where $p(\pi_{t-1}|\pi^{t-2}, \bar{d}_{t-1}, v_{t-1}, \hat{\phi})$ is the conditional inflation's density computed like in Appendix B.1. Sims et al. (2006) propose the following algorithm:

1. Suppose for every $\{\bar{d},\, v\}$ that $P[d_0 = \bar{d}, v_0 = v|\pi_0, \hat{\phi}] = \frac{1}{D \times V}$.[67]
2. For each $t = 1, 2, \ldots, T$:
   - Compute $P[\bar{d}_t = \bar{d}, v_t = v|\pi^{t-1}, \hat{\phi}]$ for every $\{\bar{d}, v\}$ using (21).
   - If $t \geq 2$, compute $P[\bar{d}_t = \bar{d}, v_t = v|\pi^t, \hat{\phi}]$ for each $\{\bar{d}, v\}$ according to (23).

Second Algorithm

The following algorithm is useful to compute the probability that the deficit's hidden state at time t is $\{\bar{d}, v\}$ given the entire history of observed inflation π^T, $P[\bar{d}_t = \bar{d}, v_t = v|\pi^T, \hat{\phi}]$. This algorithm proposed by Sims et al. (2006) is a backward recursive method that also uses Bayes Theorem. The Law of Total Probability implies that:

$$P[\bar{d}_t = \bar{d}, v_t = v|\pi^T, \hat{\phi}] = \sum_{\bar{d}_{t+1}} \sum_{v_{t+1}} P[\bar{d}_t = \bar{d}, v_t = v|\bar{d}_{t+1}, v_{t+1}, \pi^T, \hat{\phi}] P[\bar{d}_{t+1}, v_{t+1}|\pi^T, \hat{\phi}]. \tag{24}$$

To compute $P[\bar{d}_t = \bar{d}, v_t = v|\bar{d}_{t+1}, v_{t+1}, \pi^T, \hat{\phi}]$, Sims et al. (2006) propose the next equation based on Bayes Theorem:[68]

$$P[\bar{d}_t = \bar{d}, v_t = v|\bar{d}_{t+1}, v_{t+1}, \pi^T, \hat{\phi}] = \frac{Q_d(\bar{d}, \bar{d}_{t+1}) Q_v(v, v_{t+1}) P[\bar{d}_t = \bar{d}, v_t = v|\pi^t, \hat{\phi}]}{P[\bar{d}_{t+1}, v_{t+1}|\pi^t, \hat{\phi}]}. \tag{25}$$

The suggested algorithm is:

1. For each $\{\bar{d}, v\}$ compute $P[\bar{d}_T = \bar{d}, v_T = v|\pi^T, \hat{\phi}]$ using the algorithm proposed in Appendix B.2.
2. For every $t = T-1, T-2, \ldots, 1$:
   - Compute $P[\bar{d}_t = \bar{d}, v_t = v|\bar{d}_{t+1}, v_{t+1}, \pi^T, \hat{\phi}]$ for each $\{\bar{d}, v\}$ using (25). To update $P[\bar{d}_t = \bar{d}, v_t = v|\pi^t.\hat{\phi}], P[\bar{d}_{t+1}, v_{t+1}|\pi^t, \hat{\phi}]$ consider the algorithm proposed in Appendix B.2.
   - If $t \leq T-2$, compute $P[\bar{d}_t = \bar{d}, v_t = v|\pi^T, \hat{\phi}]$ for each $\{\bar{d}, v\}$ using (24).

66 Sims et al. (2006) use a common assumption made when handling hidden Markov models:

$$P[\bar{d}_t = \bar{d}, v_t = v|\pi^{t-1}, \hat{\phi}, d_{t-1}, v_{t-1}] = P[\bar{d}_t = \bar{d}, v_t = v|d_{t-1}, v_{t-1}].$$

67 D, V represent number of possible states for $\bar{d}, v$ respectively.

68 $Q_d(\bar{d}, \bar{d}_{t+1})$ means the probability of being in state $\bar{d}_{t+1}$ in $t+1$ given that $\bar{d}_t = \bar{d}$ according to Q_d.

B.3 Potential Numerical Problem

As shown in (19), if $1-\lambda\beta_t \approx 0$ or π_t is considerably high, then $f_{\pi_t}(\pi_t|\pi^{t-1}, \bar{d}_t, v_t)$ can be arbitrarily large. In this case the model's likelihood is unbounded. Therefore, if there are no restrictions imposed to $1-\lambda\beta_t$ or to π_t, the maximization problem that must be solved to find $\hat{\phi}$ is not well defined.

Sargent et al. (2009) imposed $1-\lambda\beta_t > 0$ and $\pi_t < \delta$ to deal with this numerical problem. Additionally, by assuming that π_t is randomly determined should any of these constraints is violated, it forces the estimation algorithm to find a feasible solution that satisfies these constraints, because, otherwise the model's likelihood is reduced. It can be shown that the likelihood of the model given that $1-\lambda\beta_t \leq 0$ or $\pi_t \geq \delta$ (equation (17)) is smaller than when inflation is determined according to (3).

C Fiscal Deficits Estimation

This appendix explains the procedure that I follow to estimate the fiscal deficit density at each t, $p(d_t|\pi^T, \hat{\phi})$, method inspired on Sims et al. (2006) and MacDonald and Zuccini (2009). According to the Law of Total Probability:

$$p(d_t|\pi^T, \hat{\phi}) = \sum_{i=1}^{D}\sum_{j=1}^{V} p(d_t|\pi^T, \hat{\phi}, \bar{d}_i, v_j)P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^T, \hat{\phi}]. \tag{26}$$

One common assumption when handling a hidden Markov process, is to assume that a realization of the observable variable (in this case d_t) is independent of any endogenous variable (π_t and β_t) if the non-observable part of the model is given ($\bar{d}_t, v_t$). Therefore:

$$p(d_t|\pi^T, \hat{\phi}, \bar{d}_i, v_j) = p(d_t|\hat{\phi}, \bar{d}_i, v_j). \tag{27}$$

Taking this property of hidden Markov processes as given, $p(d_t|\hat{\phi}, \bar{d}_i, v_j)$ can be easily computed: by assumption $\log(d_t|\bar{d}_t, v_t) \sim N(\log(\bar{d}_t), v_t)$. The challenging part of computing $p(d_t|\pi^T, \hat{\phi})$ is to obtain $P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^T, \hat{\phi}]$, that is, the probability that in period t the state of nature was $\{\bar{d}_i, v_j\}$ given the inflation data π^T. To compute this probability I followed the backward recursive method proposed by Sims et al. (2006), method described in Appendix B.2.

Analysing (26), $p(d_t|\pi^T, \hat{\phi})$ could be interpreted as the weighted average of the $D \times V$ possible deficit distributions.[69] Because of (27), these distributions are the same for every t. However, the assigned weight to each distribution is $P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^T, \hat{\phi}]$ and this probability will not be the same at each t.

Hence, I consider the following heuristic algorithm to estimate $p(d_t|\pi^T, \hat{\phi})$:

1. Compute $P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^T, \hat{\phi}]$ for each $i = 1, \ldots, D$, and $j = 1, \ldots, V$ following the algorithm proposed by Sims et al. (2006).

2. Simulate $\{\bar{d}_t, v_t\}$ according to $P[\bar{d}_t = \bar{d}_i, v_t = v_j|\pi^T, \hat{\phi}]$. To do this, I followed the cumulative distribution algorithm.[70]

69 With the assumptions made for D, V there are only six hidden states that determine the mean deficit $\bar{d}$ and v.

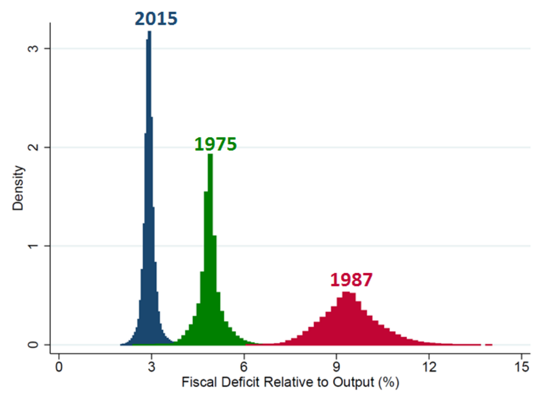
70 The Cumulative Distribution Algorithm works as follows: let $s_1, s_2, \ldots, s_M$ be all the hidden states in the model (in this case there are 6 hidden states and s_n is a vector whose elements are $\{\bar{d}_i, v_j\}$ for some i, j). Let $F_t(n)$ be the cumulative distribution function constructed as follows: $F_t(n) = \sum_{j\leq n} P[s_t = s_j|\pi^T, \hat{\phi}]$ Then, generate a pseudo-random number U following a uniform distribution on $[0, 1]$. The simulated hidden state is $s_t = s_n$ if n satisfies $F_t(n-1) < U \leq F_t(n)$.

3. Given $\{\bar{d}_t, v_t\}$, generate a pseudo-random value d_t according to the distribution $\log(d_t|\bar{d}_t, v_t) \sim N(\log(\bar{d}_t), v_t)$.

4. Repeat steps (2) and (3) N times, considering a large N. The set of simulated d_t will approximate $p(d_t|\pi^T, \hat{\phi})$ according to the Law of Large Numbers.

To give an example of how this method works, Figure IX plots the estimated annual fiscal deficit density for three different years: 1975, 1987 and 2015.[71] Considering the inflation history described in Section 3, one may conclude that the model predicts a deficit distribution with an elevated mean and variance for those years in whose inflation was elevated, like in 1987 (year characterized by having the highest inflation of the second half of the twentieth century). Whenever inflation was moderately high, like in 1975, the model predicts that fiscal deficit had a moderate mean and lower variance than in 1987. Finally, whenever inflation was controlled, the model estimates that the fiscal deficit density had a low mean and variance.

Figure IX: Fiscal Deficit Densities.



Source: Banco de México, INEGI, and SHCP.

Notes: these densities were estimated with the heuristic algorithm described in this section and $N = 100,000$ simulations.

D Inflation Predictions Procedure

To produce inflation predictions according to the model, I need a sequence of fiscal deficits relative to output for each month between January 2017 and December 2021. Consequently, I need to predict the deficit's density $p(d_{T+s}|\pi^T, \hat{\phi})$ where T represents December 2016 (the last month used to estimate the model) and $s = 1, 2, \ldots, 120$. Due to the Law of Total Probability:

$$p(d_{T+s}|\pi^T, \hat{\phi}) = \sum_{i=1}^{D} \sum_{j=1}^{V} p(d_{T+s}|\hat{\phi}, \bar{d}_{T+s} = \bar{d}_i, v_{T+s} = v_j) P[\bar{d}_{T+s} = \bar{d}_i, v_{T+s} = v_j|\pi^T, \hat{\phi}]. \quad (28)$$

As detailed in Appendix C, to estimate this density the challenging part is to compute $P[\bar{d}_{T+s} = \bar{d}_i, v_{T+s} = v_j|\pi^T, \hat{\phi}]$. Since there is no available data on future inflation, this probability cannot be computed as shown in Appendix B.2. Thus, I have to make an assumption on the way this probability changes at each $T + s$. I propose the following algorithm:

71 The density $d_t|\pi^T, \hat{\phi}$ represents the fiscal deficit distribution for each month t. The annual fiscal deficit of year Y, $d_Y|\pi^T, \hat{\phi}$, is constructed as $d_Y|\pi^T, \hat{\phi} = \sum_{t=t_1}^{t_{12}} d_t|\pi^T, \hat{\phi}$, where $t_1, \ldots, t_{12}$ are the months of year Y.

1. For each scenario, if it is assumed that fiscal deficit will have by December 2021 a median $\bar{d}^{\star}$ and variance $v^{\star}$, consider:

$$P[\bar{d}_{T+120} = \bar{d}^{\star}, v_{T+120} = v^{\star}|\pi^T, \hat{\phi}] = 1.$$

   Thus, I am assuming that in December 2021 the median deficit will be $\bar{d}^{\star}$ and the variance parameter $v^{\star}$ with probability one.

2. To compute $P[\bar{d}_{T+s} = \bar{d}, v_{T+s} = v|\pi^T, \hat{\phi}]$ for $s = 119, \ldots, 1$, consider the following backward recursive method derived from the Law of Total Probability:

$$P[\bar{d}_{T+s} = \bar{d}, v_{T+s} = v|\pi^T, \hat{\phi}] =$$

$$\sum_{i=1}^{D}\sum_{j=1}^{V} P[\bar{d}_{T+s} = \bar{d}, v_{T+s} = v|\bar{d}_{T+s+1} = \bar{d}_i, v_{T+s+1} = v_j]P[\bar{d}_{T+s+1} = \bar{d}_j, v_{T+s+1} = v_j|\pi^T, \hat{\phi}].$$

3. To compute $P[\bar{d}_{T+s} = \bar{d}, v_{T+s} = v|\bar{d}_{T+s+1} = \bar{d}_j, v_{T+s+1} = v_j]$ I use the estimated transition matrices Q_d, Q_v, and Bayes Theorem.

Once $P[\bar{d}_{T+s} = \bar{d}_i, v_{T+s} = v_j|\pi^T, \hat{\phi}]$ is computed for all $s = 1, \ldots, 120$, the fiscal deficit density at each s may be estimated following the same procedure explained in Appendix C. Then, I consider each percentile p of $p(d_{T+s}|\pi^T, \hat{\phi})$ as a possible sequence of fiscal deficits $(d^p_{T+s})_{s=1}^{120}$. Finally, considering each fiscal deficit sequence I compute the observed and expected inflation rates sequence, $(\pi^p_{T+s}, \beta^p_{T+s})_{s=1}^{120}$, generating a prediction for inflation, expectations, and fiscal deficits between 2017 and 2021.